Outsourcing, Structure of Firms, and Wage Inequality in the Market for Professional Services

Jasmin Kantarevic*
University of Toronto

August 8, 2004

Abstract

We view outsourcing as a relationship between firms that requires communication and coordination (management). Some workers have comparative advantage in management, while others have comparative advantage in the production of outsourcing services. Production skills of workers are observable, but there is symmetric learning about management skills that varies by employment sector. Labour supply decisions of workers, in addition to the outsourcing decisions of firms, determine the equilibrium extent of outsourcing, the size and hierarchical structure of firms, and the wage inequality. We show that these equilibrium outcomes tend to be positively correlated with each other, and that they vary systematically with the market size. We examine these propositions empirically using the data on the market for legal services in the U.S. and find strong support for the model. The model can be used to interpret recent increases in both the extent of outsourcing and the wage inequality and attendant changes in the structure of firms.

*jkantare@chass.utoronto.ca. I thank Arthur Hosios and Aloysius Siow for their constant encouragement and many valuable comments. I also thank participants of seminars at University of Toronto, Victoria University and York University. All remaining errors are mine.

1 Introduction

Almost every firm has to decide what to produce in-house and what to purchase from the market (outsource). The outcome of this decision determines the boundaries of the firm, an issue that has been of keen interest to economists since at least Coase (1932). Not surprisingly, there have been numerous theoretical and empirical studies examining the determinants of outsourcing decisions of firms and the variation in the extent of outsourcing across industries and markets[1].

In recent years, there has been renewed interest in the subject because of the spectacular growth in the extent of outsourcing over the last two decades. However, the emphasis in many studies has shifted from analyzing the determinants of outsourcing to understanding the impact of outsourcing on other important phenomena such as employment, wage structure, and income inequality. For example, some researchers view the rise in the extent of outsourcing as an important determinant of the recent increase in the wage inequality[2]. While such efforts are pivotal to improving our understanding of the link between the extent of outsourcing and the wage inequality, the observed correlation is open to alternative interpretations. Specifically, it is not clear whether this correlation is causal or whether changes in both the extent of outsourcing and the wage inequality can be attributed to changes in some other factors such as the market size.

What is needed to address this concern is a conceptual framework in which the extent of outsourcing is jointly determined with other phenomena of interest. This paper makes first few steps in this direction. In particular, we analyze the equilibrium process by which the extent of outsourcing, the wage inequality, and the structure of firms are jointly determined. In addition, we also study the equilibrium relation of these variables with each other, and their dependency on the market size.

The main conceptual innovation of the present paper is that we augment an equilibrium model of outsourcing with a model of labour supply of workers in the following way . We argue that the relationship between buyer firms, who are potential demanders of outsourcing services, and seller firms, who are potential suppliers of these services, involves more than simply exchanging services in the market. The buyer and seller firms have also to communicate

[1] Excellent surveys of this literature are provided by Perry (1988) and McMillan (1995)

[2] See for example Dube and Kaplan (2001).

with each other and to coordinate their efforts in order to successfully complete outsourcing projects (a function called management hereafter). This assumption creates two types of jobs in the economy: production and management. Some workers have comparative advantage in management, while others have comparative advantage in production. We assume that the information about management skills of workers is imperfect at the time workers first enter the labor market, but the market participants learn about these skills as the workers accumulate experience. In addition, management skills are valued relatively more in the seller firms because they service several buyer firms, while each buyer firm cares about completing its own outsourcing project only. For the same reason, the rate of learning about management skills is relatively faster in the seller firms.

Given this environment, the optimal behavior of buyers, sellers and workers and their interaction at the market level determine the prices and quantities in the model: the prices of outsourcing services, the wage structure, the extent of outsourcing, and the structure of seller firms: their number, employment size and the ratio of production workers to managers (leverage ratio).

We show that in the equilibrium the extent of outsourcing and the wage inequality are positively correlated. In addition, both of these variables tend to be positively associated with the employment size of seller firms, and inversely associated with the leverage ratio.

The model also delivers predictions about the relationship of these variables with the market size. This relationship depends on whether the number of workers in the market increases at a increasing or a decreasing rate with the market size. In the first case, the extent of outsourcing, the wage inequality, and the employment size of seller firms all vary directly with the market size, while the leverage ratio varies inversely with the market size. In the second case, the predictions of the model are exactly the opposite.

In the empirical part of the paper, we use data on the market for lawyers from the 1992 Economic Census and the 1990 Census of Population and Housing to evaluate these predictions. We find strong support for the model with respect to both pair-wise correlations between endogenous variables and their relation with the market size.

The remainder of the paper is organized as follows. Section 2 reviews related literature in detail and delineates the contributions of the present study. In section 3, we analyze a model in which workers have no role in the outsourcing process. In this section, we assume that buyer and seller firms can hire workers at the prevailing and exogenously determined wages. In Section 4, we enrich this model by introducing a model of labour supply of workers, analyze the optimal behavior of buyers, sellers and workers, and define an outsourcing equilibrium. In section 5, we discuss the existence and uniqueness of the outsourcing equilibrium. We also identify the conditions for the existence of two types of 'corner' equilibria that may arise in the model; namely, the no outsourcing equilibrium, in which the buyer firms perform all outsourcing services in-house, and the complete outsourcing equilibrium, in which the buyer firms outsource all tasks to the seller firms. Section 6 discusses the comparative static results of the model. Section 7 describes the data and outlines the methods for testing our empirical predictions, and section 8 presents the results of the empirical tests. We conclude in section 9.

In the conclusion of this introductory section, we wish to emphasize that our model of outsourcing is expected to fit markets for professional services, such as accounting and law, better than markets for other services. The reason is that there is a clear distinction between production workers and managers in this market and there is empirical evidence of learning about initially unobserved managerial skill in the seller firms. For example, lawyers in private law firms are usually divided into associates (junior lawyers) and partners (senior lawyer), where the first group predominantly specializes in production of legal services and the latter group predominantly specializes in dealing with clients. Promotion of associates to partners is uncertain and usually takes several years as law firms learn about the potential of associates in the partner positions.

2 Review of Related Literature

The objective of this section is to review literature on the extent of outsourcing, the wage inequality and the structure of firms and to delineate the contributions of the present study to this literature.

In this review, we focus mainly on studies that analyze outsourcing and on studies that link outsourcing to wage inequality and structure of firms. Our review is brief and we emphasize the most relevant studies only; the interested reader may find more references in the sources cited in this section and in the bibliography section at the end of this paper.

Outsourcing

There is large theoretical and empirical literature on vertical integration of firms and outsourcing. Excellent surveys are provided by Perry (1988) and McMillan (1995). Grossman and Helpman (2002) synthesize much of the previous literature in an equilibrium framework.

For the purposes of our paper, three features of this literature are particularly relevant. First, many studies have emphasized some form of market failure as a potential determinant of outsourcing. Examples include monopolistic markets for final goods, imperfect and asymmetric information, and transaction costs due to incomplete contracting. Second, most studies have analyzed outsourcing as a bilateral relationship between a single buyer firm and a single seller firm. It was only recently that Grossman and Helpman (2002) studied outsourcing in an equilibrium framework with many firms of each type. Third, and to the best of our knowledge, none of the studies examined the supply decision of workers within a model of outsourcing.

In comparison to this literature, we analyze outsourcing within a neoclassical framework with competitive markets, perfect information and no transaction costs. Our claim is not that these factors are not important; rather, we simplify this side of the problem to gain better understanding of the importance of other factors in the outsourcing process. In particular, we explicitly introduce a model of labour supply of workers to analyze the process by which changes the labour market conditions influence the extent of outsourcing. Therefore, our model simplifies the relationship between buyer and seller firms, but gives an explicit role to workers in the outsourcing process. Consistent with recent contribution of Grossman and Helpman (2002), we also study outsourcing in an equilibrium framework.

Our model delivers reduced form predictions about the relation between the extent of outsourcing and the market size that has received much attention in the previous literature. This literature has advanced a number of explanations for this relation. The most common explanation is that the extent of outsourcing increases in larger markets because firms may take advantage of economies of scale and learning and because there are more opportunities for specialization in larger markets. Some studies also argue that larger demand in big markets attracts entry of new seller firms, which enhances competition and reduces costs of outsourcing. In Grossman and Helpman (2002), the market size is measured by the size of labor force and their model predicts that the extent of outsourcing increases in bigger markets because of larger demand for final goods[3]. In our model, the market size also affects the extent of outsourcing through larger demand for outsourcing goods. In addition, the market size in our model also influences the extent of outsourcing indirectly through its impact on wages.

The empirical literature tends to find positive relationship between the extent of outsourcing and the market size. For example, Abraham and Taylor (1996), using the data from Industrial Wage Survey conducted by the Bureau of Labor Statistics between 1986 and 1987, find that the propensity of firms to outsource computer and accounting services is larger in metropolitan areas in comparison to non-metropolitan areas, but these results do not hold for janitorial, machine maintenance, and engineering and drafting services. Ono (2001) uses data from the 1992 Annual Survey of Manufacturers and finds that the probability of outsourcing for white-collar services (advertising, bookkeeping and accounting, software and data processing, and legal services) increases with an index of potential demand, but she finds no such evidence for blue-collar services (building repair, machinery repair, refuse removal).

Our study contributes to this literature by providing new empirical evidence on the relationship between the extent of outsourcing and the market size in the market for white-collar (legal) services. Our data set also allows us to study outsourcing at more disaggregated level than was possible in the previous studies. In particular, we examine the extent of outsourcing by banks and depository institutions, insurance companies, and real estate companies.

Wage Inequality

In the words of Katz and Autor, "studies of the wage structure are as old as the economics profession". In their excellent 1999 survey, they summarize evidence from many of these studies and also present a framework that attributes changes in wage structure and earnings inequality to changes in demand and supply factors and institutions.

More closely related to our paper are studies that examine link between outsourcing and wage inequality. These studies can be divided into those that look into the effect of outsourcing on wage inequality and those that look into the effect of wage inequality on outsourcing. For example,

[3] This results holds only when the matching technology is increasing returns to scale. When the matching technology is constant returns to scale, the extent of outsourcing is independent of the market size.

Abraham and Taylor (1996) find that the propensity to outsource janitorial services is higher for firms that pay high wages . They interpret this result as evidence that firms outsource in part to save on their labour costs. On the other hand, Dube and Kaplan (2001) use the CPS data between 1993 and 2000 and document that janitors and security guards employed by Building Service and Protective Service Contractors (sellers in our terminology) receive lower wages compared to janitors and guards who work for other employers.

Our model also has implications for the relation between wage inequality and the extent of outsourcing, but both of these variables are endogenously determined in our model. In our empirical analysis, we present evidence on the positive correlation between the extent of outsourcing and the wage inequality, and also show that both of these variables are systematically related to the market size.

Our paper also provides a new explanation for the wage differential between workers employed in buyer and seller firms. The previous literature has explained this wage differential in many ways. For example, the wage differential may arise due to differential unionization rates in the buyer and seller firms (Abraham and Taylor (1996)). The buyer firms may also pay higher wages to their internal workers, known in the literature as the 'efficiency' wages, to provide incentives to work harder, to reduce turnover, and to attract better applicants. The wage differential may also arise if the buyer and seller firms belong to different industries (the so called 'inter-industry' wage premium, analyzed in detail by Krueger and Summers (1987)) or because of differences in the employment size between these two types of firms (the so called 'employer size' effect, analyzed by Brown and Medoff (1985)). In addition, the wage differential may arise because of differences between the buyer and seller firms in the extent to which they are subject to various legislations regulating hiring and firing of workers. For example, Autor (2001) presents empirical evidence that the tightening of the employment-at-will regulations has raised costs of direct employment and thus led to a rise in the use of temporary help services. As a result, workers employed in buyer and seller firms may receive different wages to compensate for differences in the risk of losing jobs.

In our model, the wage differential between workers employed in the buyer and seller firms arises because jobs in these two types of firms differ in the extent to which they provide opportunities for accumulation of human capital and future career advancement. In other words, our model builds on the compensating wage differential literature (Rosen (198?)) and is determined endogenously by the demand and supply factors. The institutional reasons for the wage differential, such as unionization, are likely to be less important in the professional services sector such as legal services on which we focus in this study.

Structure of Seller Firms

As mentioned in the introduction, our model is expected to fit well in the market for professional services such as accounting and law, and we review the literature related to the structure of law firms here. For the purposes of this paper, this literature can be divided into three groups: (1) studies that examine aggregate employment and wages of lawyers at the market level (e.g. Freeman (1975), Pashigian (1977), Rosen (1992)); (2) studies that analyze the mobility of lawyers between different employment sectors (e.g. Weisbrod (1983), Goddeeris (1988), Sauer (1998)); and (3) studies that analyze economics of law firms (e.g. Mc Chesney (1982), Spurr (1987), Gilson and Mnookin (1989), Carr and Mathewson (1990), Demougin and Siow (1994), and O'Flaherty and Siow (1995). The conceptual advantage of the first group of studies is that they consider both the demand and supply side of the market, but their shortcoming is that they have less to say about the mobility of lawyers and the structure of law firms. The other two groups of studies overcome this shortcoming, but they typically consider only one side of the market (the supply side in the second group, and the demand side in the third group). In this paper, we attempt to combine advantages of all three groups of studies by analyzing both the employment decisions of individual lawyers and the decisions of law firms in an equilibrium framework. In addition, our model also incorporates choices of firms demanding legal services, such as banks and insurance companies[4] .

We borrow liberally from this literature and our debt will be clear throughout the paper. In consequence, our model addresses a number of issues that have been discussed in the literature. While many of our results are consistent with the results found in the literature, the main contribution of this paper is that presents a unified framework for analyzing these issues. Specifically, our model has implications for the number of law firms in

[4] With the exception of Pashigian (1982), most of the analyses of employment of lawyers in these firms (usually called in-house lawyers) have been conducted by members of legal profession.

the market, their employment size, the promotion probability among junior lawyers (associates), the mobility of lawyers among employment sectors, and the wage differential between associates and senior lawyers (partners).

Our model of mobility of lawyers across employment sectors also builds on recent contributions by Farber and Gibbons (1996) and Gibbons, Katz, Lemieux, and Parent (2002). Farber and Gibbons (1996) analyze a model of symmetric learning about worker's skills that are not observable by employers when the worker enters the labor market. Gibbons etal. (2002) extend this model to allow returns to observable and unobservable skills to vary by the sector of employment. In this paper, we also consider a model in which workers' skills can be divided into observable and unobservable skills, the returns to these skills vary by the employment sector, and there is a symmetric learning about the unobservable skills. In addition, we allow the rate of learning about unobservable skills to vary among employment sectors.

3 A Model with Exogenous Wages

The main conceptual innovation of this paper is to explicitly model the role of workers in the outsourcing process. To gain better insight into this role, we start in this section by analyzing a model of outsourcing in which workers play no role at all. In the spirit of industrial organization literature, we view outsourcing as a relationship between two types of firms: buyer firms, who are potential demanders of outsourcing services, and seller firms, who are potential suppliers of these services. In contrast to this literature, we analyze the outsourcing problem using the neoclassical framework in which markets are competitive, information is perfect, and there are no transaction costs.

The environment is quite simple. The economy is populated by only two types of firms: buyers and sellers. The number of buyer firms is fixed at N, and the number of seller firms, denoted by S^*, is determined by a free entry condition that will be described below. All buyer firms are identical, and all seller firms are identical, so we can discuss a representative buyer firm and a representative seller firm.

The economy lasts for one period. At the beginning of the period, each buyer firm receives a single outsourcing project. The project consists of a unit measure of identical tasks and must be completed in the same period in which it is received. Tasks can be performed using labor as the only input of production. In particular, a firm needs to employ $n(x)$ workers to perform x measure of tasks. $n(.)$ is the only technology in the economy and it is available to both buyer and seller firms. We assume that $n''(.) > 0$, which is consistent with diminishing marginal product of labour.

Buyer firms can hire workers at the wage W_B, and seller firms can hire workers at the wage W_S. Both W_B and W_S are exogenously determined. Lastly, tasks are exchanged in the competitive market for tasks at the price P.

This completes the description of the environment. We now discuss the optimal behavior of firms and the market equilibrium conditions.

The problem of a representative buyer firm is to decide what measure of tasks to perform in-house (i.e. within the firm) and what measure of tasks to outsource (i.e. purchase from the seller firms) in order to minimize the costs of completing the project. Formally, the problem can be stated as:

$$\min_{0 \leq k \leq 1} C_B(k) = W_B n(k) + P(1-k)$$

The first-order necessary and sufficient condition for the interior[5] optimum is:

$$W_B n'(k^*) = P \tag{3.1}$$

At the optimal k^* the marginal cost of performing tasks in-house, $W_B n'(k^*)$, is equated with the marginal cost of purchasing tasks from the market, P.

The problem of a representative seller firm is to maximize its profits by choosing what measure of tasks to perform:.

$$\max_{q \geq 0} Pq - W_S n(q)$$

The first-order necessary and sufficient condition for the interior optimum is:

$$P = W_S n'(q^*) \tag{3.2}$$

[5] In this section, we focus on the equilibrium in which the buyer firms perform some tasks in-house, $1 > k^* > 0$, and outsource the remaining $1 - k^*$ tasks to the seller firms. We discuss the no outsourcing equilibrium, in which the buyer firms perform all tasks in-house ($k^* = 1$), and the complete outsourcing equilibrium, in which the buyer firms outsource all tasks ($k^* = 0$), in section 5.

At the optimal q^* the marginal cost of producing tasks, $W_S n'(q^*)$, is equated to the marginal revenue from selling tasks to the buyer firms, P.

Conditions (3.1) and (3.2) can be combined into:

$$W_B n'(k^*) = W_S n'(q^*)$$

The marginal cost of performing tasks in the buyer firms is equated to the marginal cost of performing tasks in the seller firms. This condition coincides with the socially optimal level of outsourcing, given the neoclassical framework with competitive markets, perfect information, and no transaction costs, and given that both types of firms have access to identical production technology.

The market for tasks clears when the demand for tasks by the buyer firms, $N(1-k^*)$, equals the supply of tasks by the seller firms, S^*q^* :

$$N(1-k^*) = S^*q^* \tag{3.3}$$

Lastly, the number of seller firms is determined by the condition that each firm attains normal economic profits. With diminishing returns to labour, the marginal costs, $W_S n'(q)$, are larger than the average costs, $W_S n(q)/q$, for any positive value of q, and the profits are always positive. To circumvent this problem, I introduce a fixed cost of entry for each seller firm, denoted by F. The number of seller firms is then determined by:

$$Pq^* - W_S n(q^*) = F \tag{3.4}$$

The competitive outsourcing equilibrium with free entry can now be defined as a vector (P^*, k^*, q^*, S^*) which satisfies equations (3.1) to (3.4). The special case with $n(x) = x^2/2$ is discussed in Appendix.

What determines the extent of outsourcing in this model? First, the incentive to outsource is stronger when the wage of workers employed by buyer firms (W_B) is higher and when the wage of workers employed by seller firms (W_S) is lower. In a special case in which $W_B = W_S$, each of the buyer and seller firms performs half of the outsourcing project. Second, the diminishing returns to labour assumption implies that it always takes fewer workers to complete the project if the project is performed by two separate teams of workers rather than by one large team[6]. All else equal, outsourcing

[6]The proof is as follows. $n'(.) > 0$ implies $n(x)/x$ is increasing in x, which then implies that $n(x)/x < n(1)/1$ for $x < 1$ and $\sum n(x) < n(1)/1 \sum x = n(1)$. In particular,

a part of the project is less costly than completing the project entirely by each buyer firm. Lastly, higher cost of entry (F) discourages outsourcing as some seller firms exit the market, reduce the supply of tasks and therefore raise the costs of outsourcing through higher equilibrium price of tasks.

The number of buyer firms (N), a measure of the market size, does not affect the extent of outsourcing in this model. The only adjustment to the increase in the market size is the entry of new seller firms. However, the neutrality of the market size is specific to the model in which wages of workers are exogenously determined. In the next section, I show how the market size indirectly influences the extent of outsourcing through its role in determining wage differentials.

4 A Model with Endogenous Wages

The basic model of the outsourcing discussed in the previous section demonstrated how the extent of outsourcing may depend on the exogenous wages at which buyer and seller firms can hire workers. Our objective in this section is to analyze how are these wages determined. We remain within the neoclassical framework by assuming perfect information, no transaction costs, and competitive markets (now also including labor markets).

The basic model of outsourcing is enriched by introducing a model of career choice of individuals. This model builds on contributions by Farber and Gibbons (1996) and Gibbons, Katz, Lemieux, and Parent (2002). In general terms, we consider a model in which workers' skills can be divided into observable and unobservable skills, the returns to these skills vary by the employment sector, there is a symmetric learning about the unobservable skills, and the rate of learning about unobservable skills to vary among employment sectors.

The more specific ideas can be summarized as follows. The outsourcing relationship between buyer and seller firms involves not only exchanging tasks in the market, but also communication and coordination (management for short). Some workers have comparative advantage in management, while others have comparative advantage in performing tasks. The information about management skills of workers is imperfect at the time workers first enter the labor market, but the market participants learn about these skills

$n(k^*) + n(1-k^*) < n(1)$ for $k^* \in (0,1)$.

as the workers accumulate experience. Seller firms in general service several buyer firms, while each buyer firm cares only about completing its own project. For this reason, management skills are valued relatively more in the seller firms and the rate of learning about management skills is relatively faster in the seller firms.

The remainder of this section develops these ideas in detail by analyzing the decision problems of workers and each of a representative buyer firm and a representative seller firm. The section concludes by describing the market equilibrium conditions in this extended model and by a formal definition of the competitive outsourcing equilibrium with free entry.

4.1 Individuals

Since learning takes time, the economy in this section lasts forever. Every period L individuals are born who live for two periods. Individuals are risk neutral and do not discount future. In each period, individuals inelastically supply one unit of labor to the employer of their choice (a buyer or a seller firm) in order to maximize the expected present value of their lifetime income.

Each individual is endowed with a two-dimensional vector of skills (A, B). Each component of the skill vector takes the value of 1 if the individual has the relevant skill and the value of 0 otherwise. A represents the skill to perform tasks and B stands for the managerial skill.

There are three important differences between A and B. First, all individuals have the skill to perform tasks, while only some individuals have the managerial skill.

Second, all market participants have perfect information about A, but they learn about B only gradually. The rate of learning about B depends on whether the individual works for a buyer or a seller firm. Specifically, individuals who work in the seller firms for one period receive a signal that reveals their B perfectly, while individuals who work in the buyer firms learn nothing about B.

The prior probability that an individual has the managerial skill is given by $\Pr[B = 1] = \mu$. μ can be interpreted as the index of all individual's characteristics positively correlated with the promotion probability, such as education, genetic ability, ambition, and social skills[7]. For concreteness, I

[7] For example, Sauer (1998) shows that other correlates of the promotion probability in law firms include the performance in law school (GPA, class rank, participation in

will refer to μ as ability. The distribution of ability in each generation is given by a time-invariant distribution function $G(.)$ with associated density $g(.)$.

The third difference between A and B is that buyer and seller firms value these skills differently. In particular, the managerial skill is not used in the buyer firms. Individuals who work in these firms perform tasks only and earn the in-house wage W_B in each period.

In contrast, workers in the seller firms perform tasks in the first period and earn the training wage W_S. During this period, the information about the managerial skill is perfectly revealed. Individuals with $B = 1$ are promoted to managerial positions in the second period and earn the managerial wage W_M. Individuals who are not promoted can either stay in the seller firms and earn W_S performing tasks, or they can move to a buyer firm. As I will show shortly, unsuccessful trainees always have an incentive to leave the seller firms. The assumption that these trainees can always find jobs in the buyer firms implies full employment in each period. For simplicity, I assume that there are no mobility costs to individuals, nor hiring or firing costs to firms.

The career choices of individuals can now be described as follows. The income stream of an individual in his first period of life (a young individual) in the buyer firms is $2W_B$. The expected income stream of a young individual in the seller firms is $W_S + \mu W_M + [1-\mu]W_B$. The marginal young worker with ability level μ^* is indifferent between working for a seller firm and working for a buyer firm, which implies that the training wage satisfies the following condition:

$$W_S = W_B - \mu^*[W_M - W_B] \tag{4.1}$$

From equation (4.1) it is easy to see that in any equilibrium in which seller firms employ some managers, it must be the case that $W_M > W_B > W_S$. If $W_B > \max\{W_M, W_S\}$, all individuals would want to work for the buyer firms only. If $W_S > W_M$, no trainee would ever want to become a manager.

The result that $W_B > W_S$, even though individuals have identical skill to perform tasks, reflects the option value of training jobs in the seller firms. These jobs offer an opportunity to learn about one's potential in a high-wage managerial position, and individuals are willing to 'invest' early in their careers by accepting lower wages. Stated alternatively, the buyer firms offer higher wages to compensate individuals for the lack of opportunities for

mout courts, graduate school prior to the law school) and family characteristics (whether a parent was attorney).

career advancement. This result closely resembles Rosen's (1972) result that workers in jobs with higher potential to accumulate human capital are willing to accept greater reduction in their wages early in their careers to increase their earnings in the future. In my model, the form of human capital is learning about one's managerial skill.

The sorting of young individuals between buyer and seller firms is based on their ability. All young individuals with $\mu \geq \mu^*$ work for the seller firms, while all young individuals with $\mu < \mu^*$ work for the buyer firms.

Individuals in their second period of life (old individuals) who worked for the seller firms and who were promoted choose to stay in the seller firms because $W_M > W_B$. All other old individuals work for the buyer firms. In particular, unsuccessful trainees voluntarily choose to leave the seller firms because $W_B > W_S$.

4.2 Seller Firms

In this extended model, the seller firms live forever and employ two types of workers: trainees and managers. Trainees specialize in performing tasks, while managers specialize in providing client services (i.e. communication, coordination, etc.). To capture this distinction between the role of trainees and managers in a simple way, decompose the total measure of tasks performed by seller firms as $q = B \bullet k_S$, where B is the number of clients (i.e. buyer firms) each seller firm services, and k_S is the measure of tasks performed for each client.

The costs of servicing B clients and performing k_S tasks for each client can be represented as $W_S n(Bk_S) + W_M m(B)$. W_S is the training wage, W_M is the managerial wage, $n(.)$ is the number of trainees, and $m(.)$ is the number of managers. $n(.)$ has identical properties as in the previous section. $m(.)$ summarizes the management technology. Similar to $n(.)$, I assume that $m''(.) > 0$, consistent with diminishing returns to labor in management.

For the purpose of analytical simplification, I assume that seller firms set a separate team of trainees for each client they service. With this assumption, each project in the economy is completed by two teams of workers: a team of trainees in the seller firms and a team of in-house workers in the buyer firms. These two teams are identical except for the wage rate at which workers in each team can be hired. This assumption also allows a complementary interpretation of what managers in this model do. Since each seller firm services B clients, and each client is serviced by one team of trainees, the communication and coordination function of managers is closely related to their function of managing and supervising teams of trainees.

The analytical advantage of this assumption arises because the production and management decisions of seller firms can be separated into two stages. In the first stage, the seller firms decide what measure of tasks to perform for each client, and in the second stage, they decide how many clients to service. With the separability of the production and management decisions, the cost function becomes $W_S B n(k_S) + W_M m(B)$.

In this model, outsourcing is similar to a tied-in sale, because the buyer firms cannot purchase tasks without also purchasing client services. Each component of the tied-in sale is priced competitively. Specifically, the market for tasks determines the unit price of tasks P_K, as in the previous section, and the market for client services establishes the client fee P_B. For any arbitrary number of clients B and any measure of tasks for each client k_S, the revenues of seller firms are equal to $B(P_K k_S + P_B)$.

A representative seller firm now maximizes its profits by deciding in each period[8] on how many clients to service and what measure of tasks to perform for each client. Formally, the problem is:

$$\max_{0 \leq k_S \leq 1,\ B \geq 0} \Pi(k_S, B) = B[P_K k_S - W_S n(k_S) + P_B] - W_M m(B)$$

The first-order necessary and sufficient conditions for the interior[9] solution are:

$$\begin{aligned} W_S n'(k_S^*) &= P_K && (4.2) \\ P_K k_S^* - W_S n(k_S^*) + P_B &= W_M m'(B^*) && (4.3) \end{aligned}$$

Equation (4.2) is similar to the first-order condition in the basic model: at the optimal k_S^*, the marginal cost of performing an additional task, $W_S n'(k_S^*)$, is equal to its marginal revenue, P. An important property of the solution

[8] That is, we assume that tasks cannot be stored and in each period the seller firms face identical problem. This assumption excludes the possibility of strategic behaviour over time, but is adopted for analytical simplification.

[9] As in the previous section, we focus on the equilibrium in which buyer firms outsource some tasks and perform some task inhouse. The next section discusses other types of equilibria in the model.

is that k_S^* depends only on the training wage and the price of tasks and is independent of the managerial wage and the client fee. This result is due to the assumption that the production decision can be separated from the management decision. Equation (4.3) states that at the optimal B^*, the marginal profit (the left-hand side) and the marginal cost, $W_M m'(B^*)$, are set equal to each other.

Lastly, the number of seller firms is determined as in the previous section by the condition that each seller firms attains normal economic profits:

$$\Pi(k_S^*, B^*) = F \tag{4.4}$$

4.3 Buyer Firms

Buyer firms also live forever. In this extended model, a representative buyer firm has to pay a fixed client fee to each seller firm from whom it purchases any tasks. The client fee is like a fixed cost of outsourcing, and the buyer firm will minimize its outsourcing costs by dealing with a single seller firm. Given the choice of how many seller firms to deal with, the problem of a representative buyer firms in each period[10] is:

$$\min_{0 \le k_B \le 1} W_B n(k_B) + P_K(1 - k_B) + P_B$$

The first-order necessary and sufficient condition for the interior solution is:

$$W_B n'(k_B^*) = P_K \tag{4.5}$$

Equation (4.5) is identical to the first-order condition for the buyer's problem in the model of the previous section: At the optimal k_B^* the marginal cost of performing tasks in-house, $W_B n'(k_B^*)$, is equated with the marginal cost of purchasing tasks from the market, P.

4.4 Equilibrium

In this model, there are two outsourcing markets (for tasks and client services) and three labour markets (for trainees, managers, and in-house work-

[10] Buyer firms face identical problem in each period due to our assumption that the outsourcing project must be completed in the same period in which it is received. Again, this assumption is for analytical simplification.

ers). The market clearing conditions are as follows.

In the outsourcing markets, the buyer firms demand services of N seller firms and each buyer firm demands $1 - k_B^*$ tasks. Seller firms supply client services to S^*B^* clients and perform k_S^* tasks for each client. The equilibrium conditions in the market for tasks and the market for client services are, respectively, :

$$\begin{aligned} 1 - k_B^* &= k_S^* && (4.6) \\ N &= B^*S^* && (4.7) \end{aligned}$$

In the labour markets, the seller firms demand $S^*B^*n(k_S^*)$ trainees and $S^*m(B^*)$ managers, while the buyer firms demand $Nn(k_B^*)$ in-house workers. To derive the supply of trainees and managers, let $\Delta(\mu^*) \equiv 1 - G(\mu^*)$ be the fraction of individuals for whom $\mu \geq \mu^*$, and let $\Lambda(\mu^*) \equiv E[\mu|\mu \geq \mu^*]$ the average probability of promotion among trainees. Using this notation, the supply of trainees is $\Delta(\mu^*)L$ and the supply of managers is $\Lambda(\mu^*)\Delta(\mu^*)L$. All other individuals work in the buyer firms.

The clearing conditions in the market for trainees, the market for in-house workers, and the market for managers are, respectively:

$$\begin{aligned} S^*B^*n(k_S^*) &= \Delta(\mu^*)L && (4.8) \\ Nn(k_B^*) &= 2L - \Delta(\mu^*)L - \Lambda(\mu^*)\Delta(\mu^*)L && (4.9) \\ S^*m(B^*) &= \Lambda(\mu^*)\Delta(\mu^*)L && (4.10) \end{aligned}$$

We conclude this section with a definition of the outsourcing equilibrium.

Definition. *The competitive outsourcing equilibrium with free entry is a vector of prices* $(P_K^*, P_B^*, W_B^*, W_S^*, W_M^*)$ *and quantities* $(k_S^*, k_B^*, B^*, S^*, \mu^*)$ *which satisfies conditions* (4.1) *to* (4.10).

5 Types of Equilibria

In this section, we discuss the existence and uniqueness of the competitive outsourcing equilibrium with free entry. We also identify the conditions for the existence of two types of 'corner' equilibria that may arise in the model, namely, the no outsourcing equilibrium, in which the buyer firms perform all tasks in-house ($k_B^* = 1$), and the complete outsourcing equilibrium, in which the buyer firms outsource all tasks to the seller firms ($k_B^* = 0$).

The strategy for proving the existence of the competitive outsourcing equilibrium with free entry consists of first showing how the system of equations (4.1) to (4.10) can be reduced to a single equation, and then analyzing the conditions under which this equation has a solution. The first step then is to combine the equilibrium conditions in the market for trainees, the market for in-house workers, and the market for client services, and then express the equilibrium in the market for tasks as follows[11]:

$$1 - k_B\left(\mu^*; \frac{L}{N}\right) - k_S\left(\mu^*; \frac{L}{N}\right) = 0 \tag{5.1}$$

In this equation, $k_B(.)$ is a set of all pairs (k_B, μ) such that the market for in-house workers clears. $k_S(.)$ is a set of all pairs of (k_S, μ) such that the market for trainees and the market for client services clear. Equation (5.1) picks μ among these pairs that is also consistent with the equilibrium in the market for tasks.

Define the left-hand side of (5.1) for any arbitrary μ as $M(\mu)$, which can be interpreted as the excess demand for tasks. The competitive outsourcing equilibrium with free entry exists if there is a $\mu^* \in (0,1)$ such that $M(\mu^*) = 0$.

M is continuous in μ. In the appendix, we also show that M is monotonically decreasing in μ, given the equilibrium condition that the in-house wage exceeds the training wage ($W_B > W_S$). Therefore, the necessary and sufficient condition for the existence of the competitive partial outsourcing equilibrium is:

$$M(0; l) > 0 > M(1; l) \tag{5.2}$$

where l is defined as L/N, the number of workers in each generation per buyer firm in the market. The interpretation of this condition is straightforward. If all individuals were allocated to the seller firms ($\mu = 0$), there would be an excess demand for tasks (i.e. teams of trainees in the seller firms would perform fewer tasks than is necessary to complete the project), and if all individuals were allocated to the buyer firms ($\mu = 1$), there would be an excess supply of tasks. Note also that when (5.2) holds, the competitive

[11]Equation (4.1) is derived explicitly in the appendix.

outsourcing equilibrium is unique because M is monotonically decreasing in μ. The competitive outsourcing equilibrium is illustrated in graph 1.

We now further explore the existence condition and discuss the 'corner'equilibria in the model. In the appendix, we show that M decreases with l. Therefore, the competitive outsourcing equilibrium with free entry can exist only if l lies in the appropriate range: $l \in (l_{\min}, l_{\max)}$, where $l_{\min}$ and $l_{\max}$ are implicitly defined by:

$$\begin{aligned} M\left(0; l_{\min}\right) &= 0 \\ M\left(1; l_{\max}\right) &= 0 \end{aligned} \tag{5.3}$$

Again, these conditions are intuitive. When there are very few workers in the market, it is impossible to complete all outsourcing projects; and when there are very many workers in the market, it is impossible to attain full employment. When $l = l_{\min}$, we have the 'no outsourcing' equilibrium. With few workers in the market, allocating any workers to the management function does not increase production of tasks, and since outsourcing is not possible without management, the only solution is to have outsourcing projects performed entirely in-house. When $l = l_{\max}$, the only possible equilibrium is the 'complete outsourcing' equilibrium.

We summarize this discussion in the following proposition.

Proposition 1 *(Types of equilibria) When $l = l_{\min}$, buyer firms perform all tasks in-house; when $l \in (l_{\min}, l_{\max)}$, buyer firms perform some tasks in-house and outsource the rest to seller firms; and when $l = l_{\max}$, the buyer firms outsource all tasks. $l_{\min}$ and $l_{\max}$ are implicitly defined in equation (5.3).*

The model does not address the situation in which l lies outside the interval $[l_{\min},l_{\max}]$. This case can be interpret as a disequilibrium situation: either there is some unemployment (when $l > l_{\max}$) or some outsourcing projects cannot be completed (when $l < l_{\min}$).

The equilibrium interpretation can be preserved if we extend the model in either of the following directions. The first is to allow outsourcing across markets. In this case, the buyer firms in markets with $l < l_{\min}$ may purchase tasks from the seller firms located in markets with $l > l_{\max}$. We leave this extension to future research.

The second case is to allow mobility of the individuals across markets, so individuals may move from the markets in which $l > l_{\max}$ to markets in which $l < l_{\min}$. In this case, the mobility of individuals ensures that l lies in the appropriate range. A simple way to capture this idea is to endogenize the measure of workers in each generation as $L = L(N)$. $L'(N)$ is likely to be positive as individuals move from the smaller to larger markets to keep l in the interval $[l_{\min}, l_{\max}]$. The sign of $L''(N)$ is less clear. If there are some restrictions on the mobility of workers, then $L''(N)$ will be negative as the increase in the market size will be offset by increasing the number of workers but at a decreasing rate. On the other hand, if there are increasing returns to workers from locating in larger markets (e.g. lower search costs), then we can expect that $L''(N)$ will be positive.

6 Comparative Statics

The endogenous variables in the can be divided into four groups: (1) the extent of outsourcing (k_B^*, k_S^*, B^*); (2) the cost of outsourcing (P_K^*, P_B^*); (3) the wage distribution (W_B^*, W_S^*, W_M^*); and (4) the number and hiearchical structure of seller firms (S^*, μ^*). The analytical comparative static results are available for all of these variables except for P_K^* and P_B^* and these results are discussed in propositions (2) to (4). We also numerically solve the model for a special case with $n(k) = k^2/2$, $m(B) = B^2/2$ and $G(\mu) = \mu$ and present a full set of comparative static results for this case.

The model includes three parameters of interest: the size of each generation of individuals, L; the number of buyer firms, N; and the fixed cost of entry, F. We present comparative results for these parameters and also discuss the results for N when there is free mobility of workers (i.e. when $L = L(N)$). All proofs are delegated to the appendix.

Proposition 2 *When there is an increase in the size of each generation of individuals (or a decrease in the number of buyer firms), then: (i) the extent of outsourcing increases, (ii) each seller firm services more clients, (iii) the wage differential between in-house workers and trainees increases, (iv) the managerial wage falls, (v) the number of seller firms in the market falls, and (vi) the promotion probability among trainees falls.*

The increase in the size of workers in each generation or a decrease in the number of buyer firms both shift the excess demand curve in graph 1 downward, and the new equilibrium level of μ falls (that is, some in-house workers are reallocated to the seller firms). The intuition is as follows. In the equilibrium, the marginal costs of performing tasks between teams of in-house workers in the buyer firms and teams of trainees in the seller firms are equalized, which can be expressed as:

$$W_B n'(k_B) = W_S n'(k_S)$$

where we omit the stars denoting the equilibrium values. Since $W_S < W_B$, the marginal product of trainees must be smaller than the marginal product of in-house workers, or $n'(k_S) > n'(k_B)$. In addition, since the marginal product is decreasing (or $n''(.) >$), reallocating an in-house worker to a training job in a seller firm (i.e. a fall in μ) will decrease the total production of in-house workers by more than it will increase the total production of trainees. In other words, the total production of tasks will fall following the reallocation of workers and the change in excess demand for tasks will be positive. Note that this result is driven by two factors: first, the wage differential between in-house workers and trainees, which arises because training jobs provide an opportunity to learn about one's managerial ability; and second, the diminishing returns property of the production technology.

Once we identify the direction of movement of workers between the buyer and seller firms in response to changes in N or L, the results in proposition (2) are easy to understand. The workers who move from the buyer firms come from the lower end of the ability distribution and the average ability of trainees (and their probability of promotion to managers) falls. Teams of trainees become larger relative to teams of inhouse workers, and the fraction of outsourcing project performed by seller firms increases. The wage differential between in-house workers and trainees also increases due to the movement of workers from the buyer to seller firms. Since seller firms employ more trainees, the supply of managers in the future periods also increases, their wages decrease and the seller firms employ more managers and service more clients. Lastly, since the number of clients has not changed and each seller firm services more clients, this implies that there are fewer seller firms in the market.

The response of endogenous variables to changes in $l = L/N$ in a special case with $n(k) = k^2/2$, $m(B) = B^2/2$ and $G(\mu) = \mu$ are presented in figure 1[12]. This figure confirms the results presented in proposition (2) and shows

[12] In this case, the equilibrium exists if the equation

$$1 - l^{1/2}\left((2(1-\mu^*)^{1/2} + (1+\mu^*)\right)) = 0$$

that an increase in the size of each generation of individuals (or a decrease in the number of buyer firms) also leads to lower price of tasks and higher client fee.

Proposition 3 *Suppose $L = L(N)$. If $L''(.) > 0$, then the comparative statics with respect to N are identical as those for L in proposition (2). When $L''(.) < 0$, the results are opposite of those for L in proposition (2). When $L''(.) = 0$, the only effect of N is the increase in the number of seller firms in the market.*

These results are also intuitive. In this case, the increase in the number of buyer firms affect endogenous variables through two channels: first, by increasing the demand for tasks, and second, by increasing the number of workers in the market. When the number of workers increases at an increasing rate with the number of buyer firms, there will be a positive change in excess demand for tasks, and conversely in the case when the number of workers increases at a decreasing rate. When the increase in the number of workers is constant, there will be no change in the excess demand and the only adjustment to the increase in the number of buyer firms will be entry of new seller firms. Note that this last case is identical to the result obtained in the model with exogenous wages.

Proposition 4 *When the fixed cost of entry increases, then (i) the managerial wage increases, (ii) the training wage increases, (iii) the in-house wage increases, and (iv) the price of tasks increases. The extent of outsourcing, the number of clients per seller firm, the number of seller firms, and the promotion probability among trainees are all independent of changes in the fixed costs of entry.*

In the special case with $n(k) = k^2/2$, $m(B) = B^2/2$ and $G(\mu) = \mu$, it can be shown that all prices and wages considered in proposition (4) are increasing linear functions of F. On the other hand, the relation between the fixed client fee and F can be either positive or negative, depending on the assumed values for L and N.

has a solution. The fixed cost of entry F is normalized to 1. The existence of equilibrium requires that $l \in [0.16, 0.25]$ and that μ^* be consistent with $W_B > W_S$ which implies that $\mu^* \in [0, 0.236]$.

7 Data and Methods

The objective of the following two sections is to empirically evaluate a subset of propositions derived from our theoretical model of outsourcing. In this section, we describe two methods to test our empirical propositions and describe the data used in the analysis. The next section then presents the detailed results of the empirical tests.

7.1 Methods

The propositions to be tested are summarized in table 1[13]. According to the model, the endogenous variables of interest should be positively correlated, independent of the sign of $L''(N)$. In addition, all endogenous variables must vary in the same direction with the market size, regardless of the exact relationship between the market size and the size of labor force. When $L''(N) > 0$, the endogenous variables will vary positively with the market size, and conversely in the case when $L''(N) < 0$. Therefore, the first method for evaluating our empirical propositions is to test whether pair-wise correlations between endogenous variables are positive and whether these variables vary in the same direction with the market size.

The second method is more rigorous and consists of evaluating the relationship between each of the endogenous variables and the market size in a multivariate regression framework. This method recognizes that variations in the endogenous variables across markets depend not only on the market size, but also on a host of other variables such as the composition of demand and the characteristics of labor force.

In particular, for each of the endogenous variables we estimate a model of the following form:

$$Y_m = \alpha_0 + \alpha_1 X_m + \alpha_2 Z_m + \varepsilon_m \tag{7.1}$$

where Y indicates the endogenous variable of interest, X is the market size, and Z includes a set of other expected determinants of Y. The variables are defined at the market level, denoted by m. ε is the error term.

[13]The empirical definitions of the endogenous variables and the market size will be discussed in futher detail below. The definitons of these variables in terms of the theoretical constructs of the model is discussed in the appendix.

According to the predictions of the model, the coefficient a1 should be of the same sign for all endogenous variables examined. In addition, $\alpha_1 > 0$ if $L''(N) > 0$, and conversely , $\alpha_1 < 0$ if $L''(N) < 0$.

Model (7.1) can be estimated using the ordinary least squares method. However, the model with the extent of outsourcing is a bit more involved because in some markets all production may be outsourced[14]. This case can be treated in two ways. First, we can estimate model (7.1) and interpret a1 as the relationship between the extent of outsourcing and the market size, given that buyer firms produce some services in-house (i.e. $\partial E[Y|Y > 0]/\partial X$). This case corresponds to the competitive outsourcing equilibrium with free entry discussed in the previous sections. Second, we can also estimate the unconditional relationship between the extent of outsourcing and the market size (i.e. $\partial E[Y]/\partial X$). In this case, we first estimate the probability that the buyer firms employ in-house workers, $\Pr(Y_m > 0)$, using any method for discrete choice models such as probit or logit. In the second step, we estimate model (7.1) as before. Using the estimates from these two models, we can then calculate the expected extent of outsourcing in all markets as $\Pr(Y_m > 0)E[Y|Y > 0]$. In our empirical analysis, we will examine both the conditional and unconditional relation between the extent of outsourcing and the market size.

7.2 Data

Our empirical analysis focuses on the market for legal services for two important reasons. First, we have developed our model based on the extensive literature that describes the structure of this market and most of theoretical constructs of our model can be easily translated into their empirical counterparts in this market. Second, the available data for this market allows us not only to examine the structure of seller firms in great detail, but also to analyze the extent of outsourcing at much more disaggregated level than was possible in the previous empirical studies.

We use three data sets: the Legal Services portion of the 1992 Economic Census, the 1990 Census of Population and Housing, and the Bureau of Economic Analysis (BEA). The 1992 Economic Census provides a wealth of information on the structure of private law firms, while the 1990 Census

[14]The other possibility that all production takes place in-house is of little empirical importance.

provides detailed information about the characteristics of lawyers. In combination, these two data sets allow us to construct all empirical measures for our endogenous variables. We use the BEA estimates to construct a variety of measures for the market size.

The Legal Services portion of the 1992 Economic Census covers all law establishments with at least $1,000 in annual revenues, which practically implies that every law office in the United States is included. In 1992, there were 151,737 law offices and 147,130 law firms, employing 435,219 lawyers and 665,480 non-lawyers. The definition of law firms and law offices is almost identical since only about 2% of law firms have more than one office.

The Census collects variety of establishment-level information about law offices. Law offices report their annual revenues by the source (individuals, businesses, and government), the annual payroll by job categories (associates, paraprofessionals, non-legal managers and other employees) and the number of individuals in each job category, including partners and sole proprietors. The unique feature of the data is that it also reports the number of lawyers specializing in different fields of law practice, such as banking law and insurance law. The data are published at the market level (where the market is defined as MSA or PMSA) in the Sources of Receipts and Revenues and Miscellaneous Subjects parts of the Census of Services publication program.

The 1990 5 % Census of Population and Housing contains detailed information about occupation and industry of individuals. For the purposes of this analysis, I have included information on all lawyers and judges who do not reside in group quarters, who are employed and who are not in school. This sample consists of 27,985 lawyers and judges. 15 % of these lawyers were employed in the government sector, 75% worked in the legal services industry and 10% worked in other private industries. Among lawyers employed in non-legal industries, about 35% worked in the banking and credit sector (8.85%), the insurance sector (17.78%), and the real estate sector (8.09%). The Census provides data at the individual level, but the estimates at the MSA/PMSA level can be obtained by using the Census sampling weights and the place of residence.

Lastly, the BEA provides estimates of the population size, the private employment, the gross personal income, and the employment in each of the main nine industries at the MSA/PMSA level.

7.3 Description of Variables

7.3.1 Endogenous variables

Our initial empirical measure of the extent of outsourcing is the ratio of associates and partners in private law firms to the in-house lawyers employed in the business sector. The main concern with this measure is that lawyers employed in the private law firms tend to service not only business clients, but also individuals and various government agencies. To circumvent this problem, I refine my empirical measure of outsourcing by exploiting the information about the field of specialization of lawyers in the 1992 Economic Census. I focus on three fields of specialization: the banking and commercial law, the insurance law, and the real estate law. Lawyers specialized in these fields service primarily banks, depository institutions, insurance companies and real estate companies[15]. However, the 1992 Economic Census does not identify whether these specialized lawyers are associates or partners, and our empirical measure of outsourcing includes both associates and partners who are specialized in these fields. The information on the number of in-house lawyers who are employed in the banking, insurance and real estate sectors is available from the 1990 Census of Population and Housing[16]. Our refined measure of the extent of outsourcing is then the number of lawyers in private law firms specialized in field j divided by the number of lawyers employed in business sector j, where j stands for all fields, banking, insurance, or real estate. Since this ratio is not well defined for markets in which there are no in-house lawyers, we employ the inverse of this measure in the empirical analysis.

The employment size of a private law firms, our second endogenous variable of interest, is calculated as the number of associate lawyers and partners divided by the number of private law firms, available from the 1992 Economic Census.

The ratio of trainees to managers is measured as the ratio of associates

[15] Some lawyers specialized in real estate law are also expected to service individual clients.

[16] The two-year difference between the Economic Census and the Census of Population should not be of great concern because temporal variation in the extent of outsourcing is typically smaller than the across-markets variation, specially considered than in our data there is only two-year difference. Nevertheless, we have examined this by inflating the number of in-house lawyers in the 1990 Census by the rate of growth of employment in the financial sector between 1990 and 1992 and found no significant changes in our results.

to partners in private law firms. This measure is also readily constructed from the 1992 Economic Census. However, the quirk of the data is that not all private law firms have both partners and associates. In particular, the professional service organizations employ only associates and therefore these law firms cannot be included in the analysis. In addition, the solo proprietorships are also excluded from the analysis because these firms usually have only a single partner[17]. Our empirical measure is therefore confined only to the law partnerships, which employed 43.72 percent of all lawyers employed in private law firms and accounted for 47.95 percent of the total revenues in 1992.

Lastly, we have considered three measures of wage inequality that have been used frequently in the previous literature: the logarithm of standard deviation of personal income, the inter-quartile (75-25) range of income, and the coefficient of variation (the standard deviation divided by the mean) of personal income.

The summary statistics for the endogenous variables are presented in table 2. Consider the extent of outsourcing first. The ratio of in-house lawyers to lawyers in private law firms in all fields of specialization is about 0.1, ranging from 0 to 1.08. This low ratio reflects the large number of market in which there are no in-house lawyers (59 markets out of 200). The ratio is slightly higher when we consider three specific fields of specialization. The largest ratio is in the insurance sector (0.48), followed by the banking sector (0.20), and then by the real estate sector (0.14). Again, there are many markets in which there are no in-house lawyers. In particular, the number of markets with no in-house lawyers in banking industry is 106, in the insurance sector 73, and in the real estate sector 114.

With respect to the structure of markets, we find that law firms are on average small, with only 2.45 associates and partners per law firm. The distribution of the employment size of seller firms is skewed to the right, with the mean exceeding the median. We also find that there are on average twice as many partners as there are associates in our sample.

Our three empirical measure of wage inequality all indicate large variation of income of lawyers within markets as well as between markets. For example, the average inter-quartile range of income is about $60,000 (in 1992 dollars), but it ranges between $12,000 in Pueblo, CO to over $150, 000 in Kenosha,

[17]Including the solo proprietorships does not qualitatively alter our results with regard to the ratio of associates to partners.

WI.

7.3.2 Independent variables

We use three empirical measures of the market size, our main independent variable, that have been frequently employed in the previous literature: the private employment, the population size, and the gross personal income. These variables are readily available from the BEA.

We have also considered a number of other control variables that are expected to influence our endogenous variables. These control variables can be divided into those intended to capture the demand for legal services by three types of agents: individuals, businesses and government, and those that are intended to describe the characteristics of lawyers. In particular, the variables capturing the demand by individuals are: the percent of individuals who owns or has bought property, the percent of population in the age group 30-45, and the percent of population who is divorced or separated. These three variables have been found in earlier studies to be good indicators for the demand of individuals for legal services. The demand by businesses is approximated by the employment share of each of nine main industries in the private employment. The demand by various level of government is controlled by an indicator for whether the market (MSA/PMSA) is a state capital. The characteristics of lawyers include the average age, percentage of lawyers who are females, percentage of lawyers who are white, and the percentage of lawyers with professional degree.

In the empirical analysis, not all of these control variables were significant predictors of the endogenous variables of interest. Specifically, the three variables intended to capture demand of individuals for legal services (the percent of individuals who owns or has bought property, the percent of population in the age group 30-45, and the percent of population who is divorced or separated) were not significant, either individually or jointly, in any of the models analyzed. Similarly, none of the characteristics of lawyers seemed to have influence on any of the endogenous variables. For this reason, these variables were excluded from further analysis.

Instead, I have included the share of receipts of private law firms received from individualsand the government. Including these measures should partly compensate for the exclusion of variables intended to capture the demand by individuals and the exclusion of government employment, since these measures are more comprehensive and have high predictive power. However, the share of receipts received from individuals and the share of receipts received from businesses were highly collinear, and only the latter was retained in the analysis.

Table 3 presents the summary statistics. The first thing to note is that our sample covers a wide range of market sizes. While the smallest market in our sample (Enid, OK) includes slightly over 56,000 people, the largest market (Los Angeles, CA) is over 9 million people. Included are 251 MSA/PMSAs, covering 44 states. About 13 percent of markets included in the sample are state capitals. With respect to the sources of revenues of law firms, the highest share is for the receipts from individuals (55%), followed by businesses (35%), and then by the government (5%). However, there are many markets in which the receipts from businesses form the major source of revenues of private law firms. On the other hand, the share of receipts from governments is never above 20 percent. The share of main industries in the private employment reflects the distribution of employment across industries for the U.S. The largest shares are in services (34%) and retail sale (16%) sectors, while the remaining sectors have shares in the vicinity of 5 % of the private employment.

8 Empirical Results

8.1 Preliminary Analysis

As our first test of the empirical propositions implied by the model, we examine the pair-wise correlations between the endogenous variables discussed in the previous section and their correlations with the market size. The results are shown in table 4.

We present the estimated correlation coefficients, the significance levels (in square brackets) and the number of observations (in parentheses) for each of the pair-wise correlations. All variables are transformed using logarithmic transformation to ease the interpretation of correlation coefficients as unconditional elasticities. The logarithmic transformation implies that the correlations of the extent of outsourcing with other variables are specific to markets in which there are some in-house lawyers. To save the space, we also present only one of empirical measures for each of the extent of outsourcing (the insurance sector only) and the wage inequality (inter-quartile range of income) .

The results are very supportive of the theoretical model. First of all, all endogenous variables are positively correlated and 5 out of 7 pair-wise correlations are significant at 10 percent significance level or better. In addition, all endogenous variables vary with the market size (measured by the private employment) in exactly the same way and all of these correlations are significant at 1 percent significance level. The results also indicate that endogenous variables vary positively with the market size, which is consistent with the case when $L''(N) > 0$; that is, there is evidence that an increase in the market size attracts lawyers at an increasing rate .

However, the major limitation of these correlation tests is that they do not control for a host of other factors that vary across markets and that may affect the endogenous variables of interest. We address this limitation in the following subsections where we examine the relationship between the market size and each of the endogenous variables in a multivariate regression framework.

8.2 Extent of Outsourcing

We start by examining the relationship between the extent of outsourcing and the market size. Given the indication that $L''(N) > 0$ from our preliminary tests, we expect the extent of outsourcing to be positively related to the market size.

The regression results are presented in table 5. All results are obtained using the ordinary least squares method with robust standard errors.

The first column presents the results for the most general definition of outsourcing, the ratio of in-house lawyers to lawyers employed in private law firms, regardless of the industry of employment or the field of specialization. The dependent variable is specified as the logarithm of this ratio, because the logarithmic transformation produced a distribution closest to the normal, based on the skewness and kurtosis tests for normality.

The coefficient on the logarithm of private employment is negative as expected, suggesting the positive relation between the extent of outsourcing and the market size. This coefficient is not precisely estimated, but this is not surprising given that all fields of specialization are included in this definition of outsourcing. The 'source of revenues' variables are not significant, either individually or jointly. In contrast, the state capital indicator is negative and significant, indicating that the extent of outsourcing increases in markets that are state capitals. In addition, the 'share of employment' variables are jointly significant. The individually significant coefficients are for the share of employment in agriculture and wholesale trade. Jointly, the independent variables included in the model explain about 25 % of the variation in the extent of outsourcing across markets.

Columns (2) to (4) present the results for the more specific sectors: banking, insurance, and real estate. In all three columns, the coefficient on the logarithm of private employment is negative as expected and statistically significant. The implied elasticity of outsourcing with respect to the market size is highest for the real estate sector (-0.52), followed by the insurance sector (-0.47), and then by the banking sector (-0.27). The 'source of revenues' variables are significant only for the real estate sector. For this sector, the increase in the share of revenues received from businesses is associated with more outsourcing. On the other hand, the coefficient on the state capital indicator is not significant for any sector. The 'share of employment' variables are significant in the insurance and real estate sectors, but not in the banking sector. The explanatory power of the independent variables in all three sectors is quite high, ranging from 38% in the insurance sector to 47% in the real estate sector.

These basic results are fairly consistent with our expectation that the extent of outsourcing increases with the market size. However, there are a number of concerns that we address next.

The first concern is that the logarithmic specification excludes markets in which no in-house lawyers are employed. The estimates presented in table 5 are thus specific to our partial outsourcing equilibrium only, but do not address the corner equilibrium of complete outsourcing. We have therefore examined the unconditional relation between the market size and the extent of outsourcing by estimating a two-part model, using the method discussed in the previous section. In this exercise, information for all markets with non-missing observations is used, which increases the sample size substantially. The results of this exercise are presented in the second panel of table 6. The standard errors are bootstrapped (using 1,000 replications) because of the two-stage nature of the estimation process. For all measures of the extent of outsourcing, the results show that the unconditional relation between the market size and the extent of outsourcing is positive and statistically significant. In addition, the estimated elasticity for the three specific sectors is quite similar (around 35 %).

We have next examined the robustness of our results to the alternative functional form of the dependent variables. The results are presented in the third panel of table 6. We have considered the square root and identity transformation[18] of the ratio of in-house lawyers to lawyers in private law firms. The results strongly confirm our earlier findings that the extent of outsourcing varies directly with the market size. Remarkably, the coefficient of the log of private employment becomes significant using these transformations for all measures of outsourcing. However, the explanatory power using the square root and identity transformations are uniformly lower than for the basic model that uses the logarithmic specification.

The fourth panel of table 6 examines two alternative measures of market size: population size and gross personal income. The coefficients on the logarithm of these measures are all negative, and they are statistically significant for models in which the extent of outsourcing is defined for specific sectors. In addition, the coefficients are similar in magnitude to those in which the market size is proxied by private employment.

The last specification tests examine the impact of influential observations. We present the results of this exercise in the fifth panel of table 6. First, we have excluded outlier observations with large standardized residuals (greater than 2 in absolute value). Second, we have omitted observations with high leverage values (greater than $(2k+2)/n$, where k is the number of variables in the model and n is the number of observations). Third, we have excluded observation with large Cook D influential statistic (greater than $4/n$) that attempts to summarize the impact of both outliers and observations with high leverage values[19]. The results indicate that our sample contains a non-negligible number of influential markets, but these markets are not solely responsible for the sign, significance and magnitude of the coefficient on the logarithm of private employment variable that we have obtained in the basic model.

Lastly, we have also estimated the model using median regression that gives less weight to the outliers and influential observations than the ordinary least squares method. The coefficient on the log of private employment is still of expected sign and of similar magnitude as in the OLS regression, but for the banking sector this coefficient is now estimated less precisely.

In sum, these results show a systematic relation between the extent of outsourcing and the market size and confirm our expectations that this relationship is positive.

[18] That is, the ratio of in-house lawyers to the lawyers in private law firms.

[19] These cutt-off values are commonly employed in the literature.

8.3 Structure of Private Law Firms

We now discuss three variables describing the structure of private law firms: their employment size and promotion probability among associates. The regression results are presented in table 7. All results are estimated using the ordinary least squares method with robust standard errors.

The first column presents the results for the size of private law firms, measured by the number of partners and associates per law firm. We expect that law firms in larger markets employ more lawyers, or that the size of law firms and the extent of the market are positively correlated. The coefficient on the logarithm of employment variable is positive and highly significant, indicating that increasing the private employment by about 134,000 is associated on average with one more lawyer per law firm. The results also indicate that law firms are larger in markets in which revenues from businesses are relatively more important. The state capital variable is not significant. However, the 'share of employment' variables are again jointly significant. Individually significant variables are shares of employment in agriculture and construction, associated with smaller law firms, and share of employment in transportation, associated with larger law firms. The independent variables of the model explain about half of the variation in the logarithm of size of private law firms.

The last column of table 7 present the results for the leverage ratio in private law firms, defined as the number of associates per partner. We expect that the leverage ratio and the market size are positively correlated.

According to this expectation, we find that the coefficient on the logarithm of private employment variable is positive and highly significant. Increasing the private employment by 1% is on average associated with an increase in the leverage ratio by 0.2 %. In addition, the leverage ratio is higher in markets in which revenues from businesses are relatively more important. The state capital variable is again not significant, but the shares of employment are jointly significant. The only individually significant variable is the share of employment in transportation, which is positively related to the leverage ratio. The explanatory power of independent variables is about one third.

We next examine the robustness of these results to a number of specification checks. The results are presented in table 8.

We start in the second panel by examining the alternative functional forms of the dependent variables. In particular, we consider the reciprocal

(1/dependent variable) and the reciprocal root (1/square root of dependent variable) transformations. After the log transformation, these two functional forms produced the closest distributions of the dependent variables to the normal distribution . For each of the three dependent variables considered, the alternative functional forms produce highly significant coefficients of the logarithm of private employment variable. The signs of these coefficients are always positive, indicating the positive relationship between the market size and our dependent variables of interest. The explanatory power of models with these transformations is uniformly smaller than for our basic model which employed logarithmic transformation.

The third panel of table 8 examines two alternative measures of market size: population size and gross personal income. The coefficients on the logarithm of these measures are all positive and highly statistically significant, and their magnitude does not deviate substantially from the model in which the market size is proxied by private employment.

The last specification tests examine the impact of influential observations on the coefficient of the logarithm of private employment. We present the results of this exercise in the fourth column of table 8. As before, we have tried excluding outlier observations with large standardized residuals, with high leverage values, and with large Cook D influential statistics. The results indicate that our results are quite robust to excluding these observations from the sample. The results obtained from the median regression further strengthen our conclusion that our estimates are not determined solely by few unusual markets.

In sum, these results confirm our expectation of the positive relationship between the employment size of firms and the ratio of associates to partners and the market size. It is also reassuring that these variables vary with the market size in exactly the same direction as the extent of outsourcing, as our model predicts.

8.4 Wage Inequality

In this last section, we examine whether the wage inequality is positively related to the market size. As discussed earlier, we employ three measures of inequality: the standard deviation of logarithm of personal income, the interquartile (75-25) range of income, and the coefficient of variation of income.

The results are presented in table 9. As before, all results are estimated using the ordinary least squares method with robust standard errors.

For each of the measure of wage inequality considered, we find the results consistent with our expectations. The coefficients on the logarithm of private employment in all three columns in table 9 are positive, indicating that wage inequality increases in larger markets. The coefficients are also highly statistically significant, except for the model in which inequality is measurd by the coefficient of variation.

Other control variables explain little of the across-market variation in the wage inequality. None of the variables are either individually or jointly statistically significant, and the explanatory power ranges between 6 and 12 percent. The low explanatory power of the model and the insignificance of other control variables should not cause too much concern for the present study, since our purpose is primarily to examine the relationship between the market size and wage inequality and not to completely explain the variation in the wage inequality across markets. In addition, the market size is plausibly exogenous to limit the concern that other variables that are omitted from the regression may bias our inference about the relation between the market size and the wage inequality.

To further examine this inference, we have conducted a number of specification tests. The results of these tests are presented in table 10. The first set of tests relate to the functional form of the dependent variable. We have considered the square root transformation and the logarithmic transformation, based on the analysis of fit of various transformations to the normal distribution. The second set of specification tests relates to using two alternative measures of the market size: population and gross personal income. The last set of tests examines the robustness of our results to the outliers and the influential observations.

The summary of these specification tests is as follows. The most robust results are for the specification in which wage inequality is measured by the 75-25 range of personal income. In all specifications, the results indicate positive and statistically significant relationship between the wage inequality and the market size.

The results for other two measures of wage inequality (the standard deviation of logarithm of personal income and the coefficient of variation of income) have also expected signs, but are not always significant. In comparison, the results using the standard deviation of logarithm of personal income are more significant relative to the results using the coefficient of variation of personal income.

In conclusion, we interpret these results as providing reasonable evidence for the systematic positive relationship between the market size and the wage inequality.

8.5 Summary of empirical results

In this section, we have tested a subset of empirical propositions implied by our model of outsourcing. Based on these results, we found little evidence to reject the implied propositions. In particular, we have found that the four endogenous variables of the model (the extent of outsourcing, the employment size of law firms, the probability of promotion among associates in law firms, and the wage inequality among lawyers) are all positively correlated, as predicted by the model. In addition, we have found that each of these variables varies positively and significantly with the market size, which is also consistent with the implications of the model. This inference is robust controlling for a host of other factors expected to influence the endogenous variables of interest, and also to a number of specification checks including the alternative functional forms and definitions of market size, exclusion of outliers and influential observations, and using alternative estimation methods.

9 Conclusions

In this paper, we have developed an equilibrium framework that addressed the question of how are the extent of outsourcing, the employment structure of firms, and the wage inequality jointly determined. We have shown that these outcomes are expected to be positively correlated, and that they vary systematically with the market size. We have also presented empirical evidence from the market for legal services that supports these propositions.

Further empirical tests of the model for other professional services such as accounting would be most useful. However, the model is not expected to fit well in the market for other services mainly because the model of labour supply of workers may not apply to these markets. Modifying the present model with a more suitable labour supply model for services such as janitorial and security services would be another fruitful extension. Comparing the predictions of this modified model with the model presented in this paper seems promising to improve our understanding of variations in the relationship between outsourcing, structure of firms, and wage inequality across industries, and their relationship with the market size.

The model presented in this paper explains changes in the extent of outsourcing, the structure of firms, and the wage inequality through two channels: (1) changes in the market size, and (2) changes in the relationship between the labour supply of workers and the market size. Our reduced form model of the relationship between the labour supply of workers and the market size is only a first step and the future research into the foundations of this relationship should be most useful. For instance, institutional factors that limit the mobility of workers across markets are expected to influence this relationship and therefore also the extent of outsourcing, the structure of firms, and the wage inequality.

Our model has also focused on the impact of one single variable, the market size, on the extent of outsourcing, the structure of firms, and the wage inequality, and there are no doubt a myriad of other factors that may influence these variables. Probably the most fruitful approach to consider influence of these other variables is to modify our neoclassical framework to include factors studied in the industrial organization literature, such as transaction costs and market imperfections. For example, the Grossman and Helpman (2002) model which synthesizes much of the literature on vertical integration and outsourcing could be augmented to introduce a suitable model of labour supply of workers. Such a model would necessarily be more complex, but the potential benefit of better understanding the extent of outsourcing, the structure of firms, and the wage inequality may outweigh the cost of increased analytical complexity.

The model has also not considered an important case of outsourcing across markets. If we define markets as countries, the model that allows outsourcing across markets would be pivotal to understanding recent changes in global outsourcing, an issue that has received an increased interest from both the public as well as the academic community in recent years. Important contributions in this field have already been made[20]; what we suggest for future research is modifying these studies to include labour supply decision of workers that will expand the set of equilibrium outcomes that can be analyzed.

[20] See for example Antras and Helpman (2004).

10 Bibliography

Abraham, Katharine G. and Taylor, Susan K. (1996) "Firms' Use of Outside Contractors: Theory and Evidence", *Journal of Labor Economics* 14(3): pp. 394-424.

Antras, Pol and Helpman, Elhanan (2004) "Global Sourcing ", *CEPR Discussion Papers*: 4170.

Autor, David H. (2003) "Outsourcing at Will: The Contribution of Unjust Dismissal Doctrine to the Growth of Employment Outsourcing", *Journal of Labor Economics* 21(1): pp. 1-42 .

Brown, Charles and Medoff, James (1989) "The Employer Size Wage Effect", *The Journal of Political Economy* 97(5): pp. 1027-1059.

Carr, Jack and Mathewson, Frank (1990) "The Economics of Law Firms: a Study in the Legal Organization of the Firm", J*ournal of Law and Economics* 33: pp. 307-330.

Demougin, Dominique and Siow, Aloysius (1994) "Careers in Ongoing Hierarchies", *American Economic Review* 84(5): pp. 1261-1277.

Dube, Arindrajit and Kaplan, Ethan D. (2002) "Outsourcing and Wages in Service Occupations", Unpublished paper, Department of Economics, University of Chicago.

Farber, Henry S; Gibbons, Robert (1996) "Learning and Wage Dynamics", *Quarterly Journal of Economics* 111(4): pp. 1007-47

Freeman, Richard B. (1975) "Legal Cobwebs: A Recursive Model of the Market for New Lawyers", *Review of Economics and Statistics* 57: pp. 171-179.

Gibbons, Robert; Katz, Lawrence F; Lemieux, Thomas; Parent, Daniel (2002) "Comparative Advantage, Learning, and Sectoral Wage Determination", *National Bureau of Economic Research*, Inc, NBER Working Papers: 8889.

Gilson, Ronald J. and Mnookin, Robert H. (1989) "Coming of Age in Corporate Law Firm: the Economics of Associate Career Patterns", *Stanford Economic Review* 41(469): pp. 567-595.

Goddeeris, John H. (1988) "Compensation Differentials and Self-selection: an Application to Lawyers", *The Journal of Political Economy* 96(2): pp. 411-428.

Grossman, Gene M. and Helpman, Elhanan (February 2002) "Integration versus Outsourcing in Industry Equilibrium", *Quarterly Journal of Economics*, pp. 85-120.

Katz, Lawrence F. and Autor, David H. (1999) "Changes in the Wage Structure and Earnings Inequality", in O. Ashenfelter and D. Card, eds., *Handbook of Labor Economics*, Volume 3 (Elsevier Science B.V.).

Krueger, Alan and Summers, Lawrence (1988) "Efficiency Wages and the Interindustry Wage Structure", *Econometrica* 56, pp. 259-293.

McChesney, Fred (1982) "Team Production, Monitoring, and Profit Sharing in Law Firms: an Alternative Hypothesis", *The Journal of Legal Studies* 11: pp. 379-393.

McMillan, John (1995) "Reorganizing Vertical Supply Relationships", in H. Siebert, ed., *Trends in Business Organization: Do Participation and Cooperation Increase Competitiveness* (Tubingen: J.C.B. Mohr).

O'Flaherty, Brendan and Siow, Aloysius. (1992) "On the Job Screening, Up or Out Rules, and Firm Growth", *The Canadian Journal of Economics* 25(2): pp. 346-369.

Ono, Yukako (2001) "Outsourcing Business Services and the Scope of Local Markets", Working paper, *Federal Reserve Bank of Chicago*.

Pashigian, Peter B. (1977) "The Market for Lawyers: The Determinants of the Demand for and Supply of Lawyers", *Journal of Law and Economics* 20: pp. 53-85.

Pashigian, Peter B. (1982) "A Theory of Prevention and Legal Defense with an Application to the Legal Costs of Companies", *Journal of Law and Economics* 25: pp. 247-270.

Perry, Martin K. (1989) "Vertical Integration: Determinants and Effects", in R. Schmalansee and R.D. Willig, eds. *Handbook of Industrial Organization* (Amsterdam: North-Holland).

Rosen, Sherwin (1972) "Learning and Labor Market Experience", *Journal of Human Resources* 7(3): pp. 326-342.

Rosen, Sherwin (1992) "The Market for Lawyers", J*ournal of Law and Economics* 35: pp. 215-246.

Sauer, Robert M. (1998) "Job Mobility and the Market for Lawyers", Journal of Political Economy 106(1): pp. 147-71.

Spurr, Stephen J. (1987) "How the Market Solves an Assignment Problem: the Matching of Lawyers with Legal Claims", *Journal of Labor Economics* 5(4): pp. 502-532.

Weisbrod, Burton A. (1983) "Nonprofit and Proprietary Sector Behavior: Wage Differentials among Lawyers", *Journal of Labor Economics* 1(3): pp. 246-263.

11 Appendix

11.1 Equilibrium with Exogenous Wages

This section solves for the outsourcing equilibrium of the model presented in section 3 in the special case with $n(x) = x^2/2$. In this case, the equilibrium is defined by the following four conditions:

$$\begin{aligned} W_B k^* &= P^* && (11.1.1) \\ W_S q^* &= P^* && (11.1.2) \\ N(K - k^*) &= S^* q^* && (11.1.3) \\ P^* q^* - W_S q^{*2}/2 &= F && (11.1.4) \end{aligned}$$

Combining equations (11.1.2) and (11.1.4), we can solve for q^* :

$$q^* = \left(\frac{2F}{W_S}\right)^{\frac{1}{2}} \tag{11.1.5}$$

In the next step, combine (11.1.1) and (11.1.2) with (11.1.5) to solve for k^* :

$$k^* = \frac{1}{W_B}\left(2FW_S\right)^{\frac{1}{2}} \tag{11.1.6}$$

P^* can be found using either (11.1.1) with (11.1.6) or using (11.1.2) with (11.1.5):

$$P^* = \left(2FW_S\right)^{\frac{1}{2}} \tag{11.1.7}$$

Lastly, S^* can be found using (11.1.4) in combination with (11.1.5) and (11.1.6):

$$S^* = \left(\frac{W_S}{2F}\right)^{\frac{1}{2}} NK - \frac{W_S}{W_B} \tag{11.1.8}$$

The comparative static results are summarized in the following table:

	W_B	W_S	N	K	F
k^*	$-$	$+$	0	0	$+$
q^*	0	$-$	0	0	$+$
P^*	0	$+$	0	0	$+$
S^*	$+$	$+$	$+$	$+$	$-$

11.2 Derivation of Eqq. (5.1)

Combine equations (4.7) and (4.8) to express the equilibrium in the market for trainees as:

$$\begin{aligned} k_S^* &= f\left(\Delta(\mu^*)l\right) \\ &= f((1-G(\mu^*))l) \\ &\equiv k_S(\mu^*;l) \end{aligned} \tag{11.2.1}$$

Here, $f(.)$ is the inverse of $n(.)$, and $l \equiv L/N$. Since $n(.)$ is strictly increasing and strictly convex, $f(.)$ exists and is strictly increasing and strictly concave.

We can solve for k_B^* directly from equation (4.9):

$$\begin{aligned} k_B^* &= f\left((2-\Delta(\mu^*)-\Delta(\mu^*)\Lambda(\mu^*)l\right) \\ &= f\left(\left(1+G(\mu^*)-\int_{\mu^*}^{1}\mu g\left(\mu\right)d\mu\right)l\right) \\ &\equiv k_B(\mu^*;l) \end{aligned} \tag{11.2.2}$$

The second line follows since $\Delta(\mu^*)\Lambda(\mu^*) = (1-G(\mu^*)E[\mu \mid \mu > \mu^*] = \int_{\mu^*}^{1}\mu g\left(\mu\right)d\mu$.

Equation (5.1) is then obtained by substituting for k_B^* and k_S^* from equations (11.2.1) and (11.2.2) into equation (4.6).

11.3 Proof that $\partial M/\partial\mu < 0$

Using equations (11.2.1) and (11.2.2), express $M(\mu;l)$ as:

$$M(\mu;l) = 1 - f\left(\left(1+G(\mu)-\int_{\mu}^{1}\tilde{\mu}g\left(\tilde{\mu}\right)d\tilde{\mu}\right)l\right) - f((1-G(\mu))l) \tag{11.3.1}$$

Differentiating with respect to μ, we have:

$$\begin{aligned}\frac{\partial M(\mu;l)}{\partial\mu} &= -lf'(n_B)g(\mu)(1+\mu)+lf'(n_S)g(\mu) \\ &= -lg(\mu)(f'(n_B)(1+\mu)-f'(n_S))\end{aligned}$$

Here, $n_B = n(k_B)$ and $n_S = n(k_S)$. Since n_B must be smaller than n_S if $W_B > W_S$, the concavity of f then implies that $\frac{\partial M(\mu;l)}{\partial\mu} < 0$.

11.4 Proof that $\partial M/\partial l < 0$

Differentiating equation (11.3.1) with respect to l we have:

$$\begin{aligned}\frac{\partial M(\mu;l)}{\partial l} &= -f'(n_B)\left(1+G(\mu)-\int_{\mu}^{1}\tilde{\mu}g(\tilde{\mu})\,d\tilde{\mu}\right)-f'(n_S)(1-G(\mu)) \\ &< 0\end{aligned}$$

where the second line follows since $f(.)$ is strictly increasing function, $G(\mu) \geq 0$, and $1 \geq \int_{\mu}^{1}\tilde{\mu}g(\tilde{\mu})\,d\tilde{\mu}$.

11.5 Proof of Proposition (2)

Since $\partial M/\partial\mu$ and $\partial M/\partial l$ are both negative, it follows from equation (11.3.1) that $\partial\mu/\,\partial l < 0$.

To find $\partial k_S/\,\partial l$, differentiate equation (11.2.1) with respect to k_S and l:

$$\partial k_S = f'(\Delta(\mu^*)l)\bullet\left(\Delta(\mu^*)-g(\mu^*)\frac{\partial\mu}{\partial l}l\right)\partial l$$

so $\partial k_S/\,\partial l > 0$ since $f' > 0, \Delta(\mu^*) > 0, g(\mu^*) > 0$ and $\partial\mu/\,\partial l < 0$.

To find $\partial B/\,\partial l$, use equation (4.7) to express (4.10) as:

$$\frac{m(B^*)}{B^*} = l\int_{\mu^*}^{1}\mu g(\mu)\,d\mu \tag{11.5.1}$$

Differentiating with respect to B and l yields:

$$\frac{m'(B^*)B^*-m(B^*)}{B^{*2}}\partial B = \left(\int_{\mu^*}^{1}\mu g(\mu)\,d\mu - l\mu^* g(\mu^*)\frac{\partial\mu}{\partial l}\right)\partial l$$

so $\partial B/\partial l > 0$ since $m'(B^*)B^* - m(B^*) > 0$ as m is convex, $\int_{\mu^*}^{1} \mu g(\mu)\, d\mu > 0, l\mu^* g(\mu^*) > 0$,and $\partial \mu / \partial l < 0$.

S^* depends on N and L separately. From equation (4.7), totally differentiating with respect to S and L, and then with respect to S and N yields:

$$\begin{aligned} \partial S &= \frac{B^* + N\frac{\partial B}{\partial l}\frac{L}{N^2}}{B^{*2}} \partial N \\ \partial S^* &= \frac{-\frac{\partial B}{\partial l}}{B^{*2}} \partial L \end{aligned}$$

so we have $\partial S/\partial N > 0$ and $\partial S/\partial L < 0$ since $\partial B/\partial l > 0$.

To find $\partial W_M/\partial l$, combine equations (4.3) and (4.4) to get:

$$W_M^* \left(m'(B^*)B^* - m(B^*)\right) = F \tag{11.5.2}$$

Differentiating with respect to W_M and l yields:

$$\left(m'(B^*)B^* - m(B^*)\right) \partial W_M^* = -W_M^* m''(B^*) \frac{\partial B}{\partial l} \partial l$$

$m'(B^*)B^* - m(B^*) > 0$ since m is convex, and $\partial B/\partial l > 0$.Therefore, $\partial W_M/\partial l < 0$.

Lastly, combine equations (4.2), (4.5) and (4.6):

$$\frac{W_B^*}{W_S^*} = \frac{n'(k_S^*)}{n'(1 - k_S^*)} \tag{11.5.3}$$

Differentiate with respect to W_B/W_S and l :

$$\partial \left(\frac{W_B}{W_S}\right) = \frac{n''(k_S^*)n'(k_B^*) + n'(k_S^*)n''(k_B^*)}{n'(k_B^*)^2} \frac{\partial k_S}{\partial l} \partial l$$

so we have $\partial \left(\frac{W_B}{W_S}\right) / \partial l > 0$ since $n', n'' > 0$ and $\frac{\partial k_S}{\partial l} > 0$.

These results are summarized in the following table:

	L	N
k_S^*	$+$	$-$
B^*	$+$	$-$
W_B^*/W_S^*	$+$	$-$
W_M^*	$-$	$+$
S^*	$-$	$+$
μ^*	$-$	$+$

11.6 Proof of Proposition (3)

When $L''(.) > 0$, then $\partial(L(N)/N)/\partial N > 0$. In this case, $\partial M(\mu; N)/\partial N < 0$. The proof then proceeds exactly as in the proof of proposition (2) treating N as $l = L/N$. In the case of $L''(.) < 0$, $\partial(L(N)/N)/\partial N < 0$ and $\partial M(\mu; N)/\partial N > 0$. The proof then follows the same lines as the proof of proposition (2) treating N as $-l$. Finally, if $L''(.) = 0$, all variables that depend only on $L(N)/N$ are not affected by changes in N. The only equation where N arises independently is (4.7) from which we have that $\partial S^*/\partial N > 0$.

.

11.7 Proof of Proposition (4)

Since F does not enter equation (11.3.1), μ^* is independent of F. In addition, k_S^*, B^*, and S^* can be expressed only in terms of μ^* and l, and therefore also do not depend on F.

$\partial W_M/\partial F$ can be found by differentiating equation (11.5.2) with respect to W_M and F :

$$(m'(B^*)B^* - m(B^*))\,\partial W_M = \partial F$$

so $\partial W_M/\partial F > 0$.

Now, combine equations (4.1), (4.2) and (4.5) to express W_M^* as:

$$W_M^* = \left(\frac{1+\mu^*}{\mu^*}\frac{n'(k_S^*)}{n'(1-k_S^*)} - \frac{1}{\mu^*}\right) W_S^*$$

Differentiating with respect to W_S and F yields:

$$\frac{\partial W_M}{\partial F}\partial F = \left(\frac{1+\mu^*}{\mu^*}\frac{n'(k_S^*)}{n'(1-k_S^*)} - \frac{1}{\mu^*}\right) \partial W_S$$

The term in the brackets is since $\frac{n'(k_S^*)}{n'(1-k_S^*)} > 1$ and $\frac{1+\mu^*}{\mu^*} > \frac{1}{\mu^*}$. Therefore, we have $\partial W_S/\partial F > 0$.

Lastly, to find $\partial P_K/\partial F$ differentiate either (4.2) or (4.5) to get:

$$\partial P_K = n'(k_S^*)\frac{\partial W_S}{\partial F}\partial F$$

so $\partial P_K/\partial F > 0$.

11.8 Empirical Predictions of the Model

1.Extent of Outsourcing

The empirical measure of the 1/extent of outsourcing can be defined in terms of the variables of the model as:

$$Y = \frac{Nn(k_B)}{SBn(k_S) + Sm(B)} = \frac{n(1-k_S)}{n(k_S) + \frac{m(B)}{B}}$$

where we used two equilibrium conditions, $k_B + k_S = 1$ and $N = SB$ to arrive at the expression at the right-hand side of the equation.

Differentiating with respect to N and Y we have:

$$\left(n(k_S) + \frac{m(B)}{B}\right)^2 dY = \left\{-n'(k_B)\left(n(k_S) + \frac{m(B)}{B}\right)\frac{\partial k_S}{\partial(L/N)}\right\}\frac{d(L/N)}{dN}dN$$
$$\left\{-n(k_B)\left(n'(k_S)\frac{\partial k_S}{\partial(L/N)} + \frac{m'(B)B - m(B)}{B^2}\frac{\partial B}{\partial(L/N)}\right)\right\}\frac{d(L/N)}{dN}dN$$

Now, $\frac{\partial k_S}{\partial(L/N)} > 0$ and $\frac{\partial B}{\partial(L/N)} > 0$ from Proposition 2. Therefore, if $\frac{d(L/N)}{dN} > 0$, then $dY/dN < 0$ (i.e. the extent of outsourcing increases with the market size), and conversely, when $\frac{d(L/N)}{dN} < 0$, then $dY/dN > 0$.

2. Employment size of seller firms

The employment size of seller firms is defined as:

$$Y = \frac{SBn(k_S) + Sm(B)}{S} = Bn(k_S) + m(B)$$

Differentiating with respect to N and Y we have:

$$dY = \left\{Bn'(k_S)\frac{\partial k_S}{\partial(L/N)} + \left(n(k_S) + m'(B)\frac{\partial B}{\partial(L/N)}\right)\right\}\frac{d(L/N)}{dN}dN$$

Since $\frac{\partial k_S}{\partial(L/N)} > 0$ and $\frac{\partial B}{\partial(L/N)} > 0$ from Proposition 2, $dY/dN > 0$ if $\frac{d(L/N)}{dN} > 0$, (i.e. the employment size of firms increases with the market size), and conversely, when $\frac{d(L/N)}{dN} < 0$, then $dY/dN < 0$.

3. Ratio of junior to senior workers

The ratio of junior to senior workers is defined as:

$$Y = \frac{SBn(k_S)}{Sm(B)} = \frac{Bn(k_S)}{m(B)} = \frac{1}{\Lambda(\mu)}$$

where the last line uses equilibrium conditions (4.8) and (4.10) and $\Lambda(\mu)$ is the average probability of promotion among trainees. Since $\Lambda'(\mu) > 0$, and $d\mu/dN < 0$ when $\frac{d(L/N)}{dN} > 0$ and $d\mu/dN > 0$ when $\frac{d(L/N)}{dN} < 0$ from Proposition (3), we have $dY/dN > 0$ if $\frac{d(L/N)}{dN} > 0$ (i.e. the ratio of junior to senior workers increases with the market size), and $dY/dN < 0$ when $\frac{d(L/N)}{dN} < 0$.

4. Wage inequality

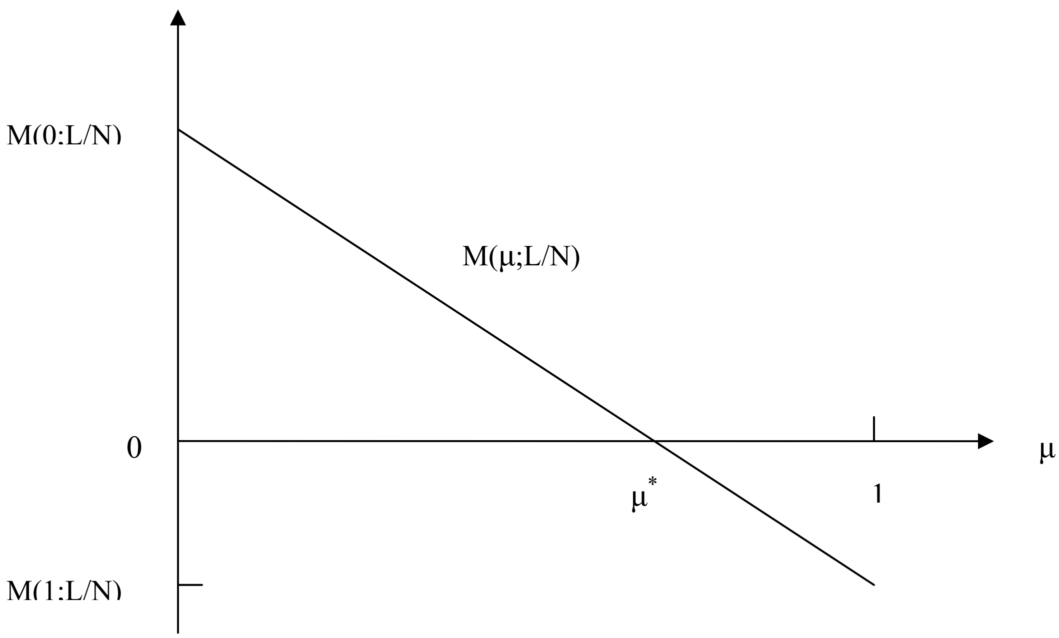
The lowest paid worker in the model is the trainee, and the best paid worker in the model is the manager. Therefore, the bigger the difference between W_S and W_M, the larger is wage inequality in the market. Using equation (4.1) we can express W_M/W_S as:

$$Y = \frac{W_M}{W_S} = \frac{W_B}{W_S}\frac{1+\mu}{\mu} - \frac{1}{\mu}$$

Differentiating with respect to N and Y we have:

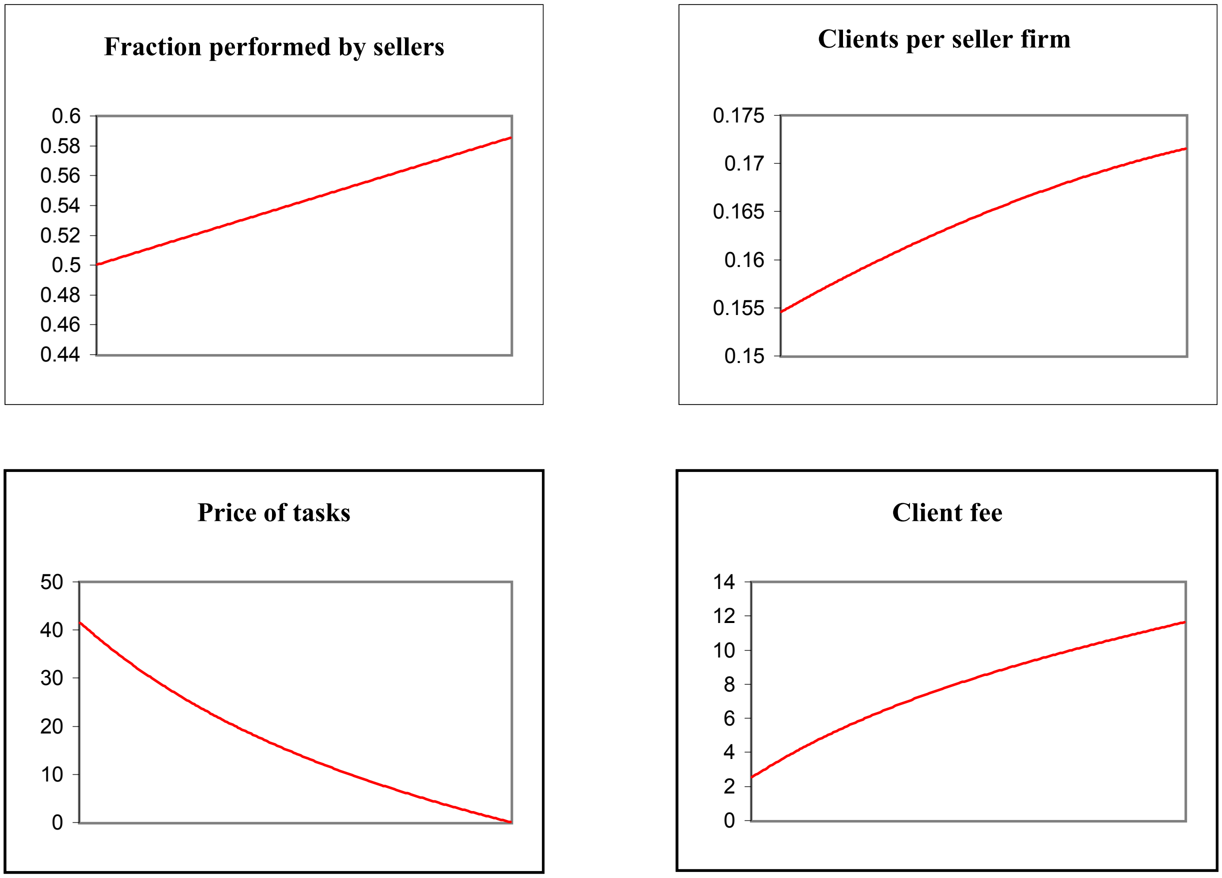
$$dY = \left\{ \frac{1+\mu}{\mu}\frac{\partial(W_M/W_S)}{\partial(L/N)} + \frac{1}{\mu^2}\left(1 - \frac{W_B}{W_S}\right)\frac{\partial\mu}{\partial(L/N)} \right\} \frac{d(L/N)}{dN} dN$$

Now, $\frac{\partial(W_M/W_S)}{\partial(L/N)} > 0$ and $\frac{\partial\mu}{\partial(L/N)} < 0$ from Proposition 2. In addition, $1 - \frac{W_B}{W_S} < 0$ because $W_B > W_S$. Therefore, the term in brackets is positive. Now, if $\frac{d(L/N)}{dN} > 0$, we have $dY/dN > 0$ (i.e. the wage inequality increases with the ,market size), and $dY/dN < 0$ when $\frac{d(L/N)}{dN} < 0$.



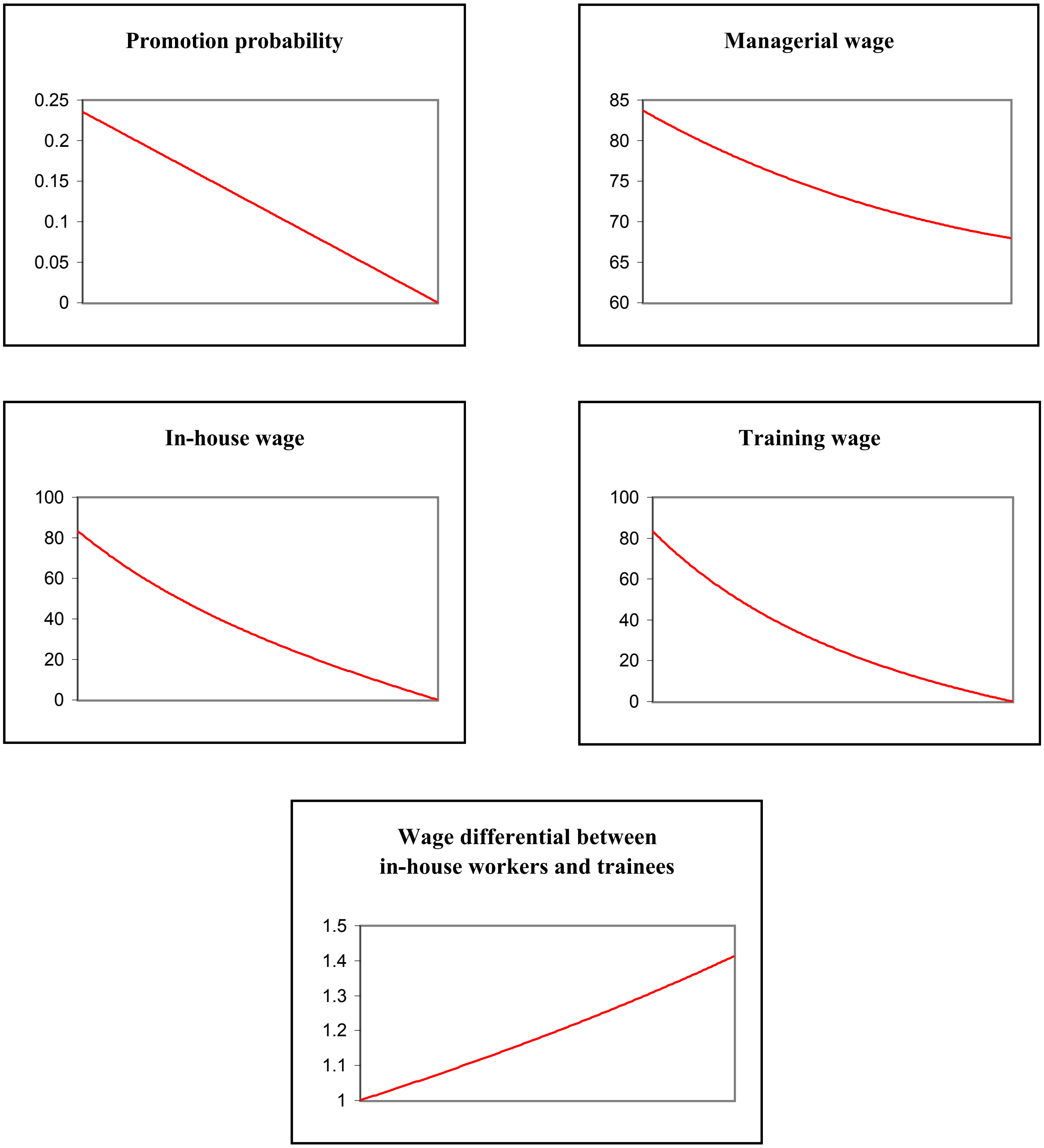
Graph 1: Competitive outsourcing equilibrium

Figure 1
Response of endogenous variables to changes in the ratio of young workers to the number of buyer firms



Note : The ratio of young workers to the number of buyer firms ranges from 0.163654 to 0.171531.

Figure 1 (continued)
Response of endogenous variables to changes in the ratio of young workers to the number of buyer firms



Note : The ratio of young workers to the number of buyer firms ranges from 0.163654 to 0.171531.

Table 1. Empirical predictions of the model

	Market size	
Endogenous variables	Case 1: $L''(N) > 0$	Case 2: $L''(N) < 0$
1. Extent of outsourcing	+	-
2. Employment size of seller firms	+	-
3.Trainee-manager ratio in seller firms	+	-
4.Wage inequality	+	-

Table 2. Summary statistics for endogenous variables

Variable	Definition	Obs.	Mean	St.Dev.	Minimum	Maximum
Extent of Outsourcing	In-house lawyers per lawyers in private law firms					
All fields		200	.1068	.1381	0	1.08
Banking		169	.1998	.4239	0	2.28
Insurance		154	.4780	.9174	0	5.25
Real Estate		163	.1357	.2732	0	1.24
Structure of law firms						
Size	Associates and partners per private law firm	247	2.45	0.71	1.18	5.41
Leverage ratio	Associates per partner in private law firms (Law partnerships only)	232	0.51	0.35	0.03	2
Wage Inequality						
Log of st.dev. of income	Logarithm of standard deviation of total income	200	-.3006	.2339	-1.09	.30
Inter-quartile range of income	Difference in income between the 75^{th} and the 25^{th} percentile	200	60,723	24,376	12,431	157,067
Coefficient of variation	Standard deviation/Mean of Total Income	200	.6928	.1382	0.33	1.25

Table 3. Summary statistics for independent variables

Variable	Definition	Obs.	Mean	St.Dev.	Minimum	Maximum
Market size						
Private employment	Employees in private sector (‘000s)	251	329.77	581.56	25.22	4,417.88
Population	Population size (‘000s)	251	703.69	1,192.8	56.78	9,055.42
Personal income	Personal income (million, 1992 $)	251	15.41	29.52	1.03	241.23
		251				
State capital	=1 if state capital		0.13	0.33	0.00	1.00
		251				
Sources of receipts						
Individuals	% of revenues from individuals	251	55.15	15.91	4.80	97.50
Businesses	% of revenues from businesses	251	35.47	14.71	2.30	70.80
Government	% of revenues from government	251	4.59	3.11	0.08	19.30
		251				
Share of employment	Share of private employment					
Agriculture and mining		251	0.02	0.03	0.01	0.21
Construction		251	0.06	0.01	0.04	0.15
Finance		251	0.08	0.02	0.05	0.21
Manufacturing		251	0.16	0.07	0.02	0.38
Retail sale		251	0.22	0.03	0.12	0.34
Services		251	0.34	0.05	0.24	0.52
Transportation		251	0.05	0.02	0.02	0.12
Wholesale trade		251	0.05	0.01	0.02	0.11

Table 4. Pairwise correlations between endogenous variables and the market size

	1/Extent of outsourcing (Insurance)	Size of law firms	Associates to partners	Inter-quartile range of income
Size of law firms	**-0.1521**			
	[0.1836]			
	(78)			
Associates to partners	**-0.4052**	**0.4112**		
	[0.0002]	[0.000]		
	(78)	(284)		
Inter-quartile range of income	**-0.1237**	**0.1711**	**0.1302**	
	[0.2871]	[0.0154]	[0.0725]	
	(76)	(200)	(191)	
Private employment	**-0.4294**	**0.5159**	**0.4752**	**0.2887**
	[0.0001]	[0.0000]	[0.0000]	[0.0000]
	(79)	(302)	(287)	(202)

All variables are in logarithms. The levels of significance are in square brackets. The numbers of observations are in parentheses.

Table 5. Regression results: Extent of outsourcing

	Dependent variable = Logarithm (In-house lawyers/Lawyers in private law firms)			
	(1)	**(2)**	**(3)**	**(4)**
	All fields	**Banking**	**Insurance**	**Real Estate**
log (employment)	-0.0278	-0.2658*	-0.4738**	-0.517***
Receipts from businesses	-0.0058	-0.0111	-0.0005	-0.0289**
Receipts from government	0.0170	-0.1123	0.0203	0.0732
State Capital	-0.3084**	-0.4120	-0.5199	-0.4248
Share agriculture	-9.39***	3.05	-9.35*	-12.61**
Share construction	-2.76	-11.45	-12.88*	3.17
Share finance	5.31	20.71**	14.92*	11.64
Share manufacturing	1.48	8.08*	4.48	2.09
Share retail trade	-3.89	18.24*	7.14	-8.97
Share transportation	2.98	9.11	3.65	-10.56
Share wholesale trade	-9.89*	-7.38	-1.87	-1.40
Number of observations	141	57	76	50
Adjusted R^2	0.2469	0.4048	0.3758	0.4722
p-value on receipts variables	0.3459	0.1570	0.9296	0.0325
p-value on shares variables	0.000	0.2226	0.0005	0.0329

*** significance at 1%, ** significance at 5%, * significance at 10%.

Table 6. Specification checks: Extent of outsourcing

	Dependent variable = Logarithm (In-house lawyers/Lawyers in private law firms)			
	All fields	**Banking**	**Insurance**	**Real Estate**
Panel 1				
Basic Model	-0.0278	-0.2658*	-0.4738**	-0.517***
(Adjusted R^2)	(0.2469)	(0.4048)	(0.3758)	(0.4722)
[Number of observations]	[141]	[57]	[76]	[50]
Panel 2				
Two-part model	-0.4836***	-0.3761***	-0.3559***	-0.3417***
(Bootstrapped standard error)	(0.0234)	(0.0270)	(0.0236)	(0.0264)
[Number of observations]	[200]	[162]	[162]	[162]
Panel 3				
Dependent variable				
Square root	-0.0277*	-0.1185**	-0.2654**	-0.1272**
(Adjusted R^2)	(0.2622)	(0.3433)	(0.4443)	(0.4034)
Identity	-0.0450**	-0.2126*	-0.6954***	-0.3984*
(Adjusted R^2)	(0.2664)	(0.2386)	(0.4554)	(0.2676)
Panel 4				
Market size:				
log (population)	-0.0293	-0.2184*	-0.4582***	-0.4550***
log (personal income)	-0.0331	-0.2279*	-0.4744***	-0.4387***
Influential observations				
Studentized residuals > 2	0.0537	-0.1624*	-0.5354**	-0.3939**
(Number of excluded obs.)	(5)	(2)	(2)	(1)
Leverage > (2k+2)/n	0.0312	-0.1541*	-0.4646***	-0.4250**
(Number of excluded obs.)	(6)	(2)	(4)	(2)
Cook's D statistic > 4/n	0.0649	-0.2762**	-0.5723***	-0.4737***
(Number of excluded obs.)	(11)	(8)	(6)	(8)
Median regression	0.0177	-0.2098	-0.5241***	-0.3334*

*** significance at 1%, ** significance at 5%, * significance at 10%.
k = number of variables, n = number of observations.

Table 7. Regression results: Structure of law firms

	Dependent variable = Logarithm of	
	(1)	**(2)**
	Size of firms	**Leverage ratio**
log (employment)	0.0491***	0.2251***
Receipts from businesses	0.0073***	0.0110***
Receipts from government	0.0053	0.0014
State Capital	0.0676	0.1801
Share agriculture and mining	-0.8061**	1.1905
Share construction	-2.1984**	1.3998
Share finance	0.9384	3.7510
Share manufacturing	0.1090	-0.4953
Share retail trade	-0.7241	0.9632
Share transportation	1.4334*	4.9748*
Share wholesale trade	0.5449	-5.8553
Number of observations	247	232
Adjusted R^2	0.5277	0.3263
p-value on receipts variables	0.0000	0.0397
p-value on shares of employment	0.0005	0.0099

*** significance at 1%, ** significance at 5%, * significance at 10%.

Table 8. Specification checks: Structure of law firms.

	Dependent variable:	
	Size of firms	**Leverage ratio**
Panel 1		
Basic Model	0.0491***	0.2251***
(Adjusted R^2)	(0.5277)	(0.3263)
[Number of observations]	[247]	[231]
Panel 2		
Dependent variable		
Reciprocal root	-0.0143***	-0.1819***
(Adjusted R^2)	(0.5115)	(0.2547)
Reciprocal	-0.0169***	-0.6445**
(Adjusted R^2)	(0.4882)	(0.1744)
Panel 3		
Market size:		
log (population)	0.0362***	0.2198***
log (personal income)	0.0415***	0.2111***
Panel 4		
Influential observations		
Studentized residuals > 2	0.0418***	0.2443***
(Number of excluded obs.)	[11]	[3]
Leverage > (2k+2)/n	0.0559***	0.2053***
(Number of excluded obs.)	[16]	[17]
Cook's D statistic > 4/n	0.0379***	0.1841***
(Number of excluded obs.)	[18]	[21]
Median regression	0.0595***	0.2185***

*** significance at 1%, ** significance at 5%, * significance at 10%.
k = number of variables, n = number of observations.

Table 9. Regression results: Wage inequality

	Dependent variable = Logarithm of		
	(1)	**(2)**	**(3)**
	St.Dev. of log income	**Coefficient of Variation**	**Interquartile range**
log (employment)	0.0565**	0.0265	0.1126***
Receipts from businesses	-0.0002	0.0009	0.0006
Receipts from government	0.0041	0.0018	-0.0012
State Capital	-0.0343	0.0262	-0.1277
Share agriculture and mining	-0.7924	-1.4277	0.8691
Share construction	0.6655	1.8122	0.3471
Share finance	-0.2235	-0.3558	-0.9448
Share manufacturing	0.1623	0.0142	0.4521
Share retail trade	0.7885	0.5186	-0.0059
Share transportation	-0.6205	-0.5422	1.5937
Share wholesale trade	-1.0333	-0.7845	2.8970
Number of observations	200	200	200
Adjusted R^2	0.0561	0.0854	0.1153
p-value on receipts variables	0.7511	0.7930	0.9703
p-value on shares of employment	0.7295	0.1848	0.6237

*** significance at 1%, ** significance at 5%, * significance at 10%.

Table 10. Specification checks: Wage Inequality.

	Dependent variable:		
	St.Dev. of log income	**Coefficient of Variation**	**Interquartile range**
Basic Model	0.0565**	0.0265	0.1126***
(Adjusted R^2)	(0.0561)	(0.0854)	(0.1153)
[Number of observations]	[200]	[200]	[200]
Dependent variable			
Square root	0.0199***	0.0076	11.5747***
(Adjusted R^2)	(0.0421)	(0.0720)	(0.1000)
Identity	0.0272*	0.0072	4778.51**
(Adjusted R^2)	(0.0310)	(0.0607)	(0.0921)
Market size:			
log (population)	0.0521***	0.0220	0.1036***
log (personal income)	0.0478***	0.0203	0.0965***
Influential observations			
Studentized residuals > 2	0.0838***	0.0371*	0.1288***
(Number of excluded obs.)	[8]	[6]	[5]
Leverage > (2k+2)/n	0.0357	0.0142	0.1094***
(Number of excluded obs.)	[11]	[9]	[9]
Cook's D statistic > 4/n	0.0377*	0.0303*	0.0989***
(Number of excluded obs.)	[16]	[17]	[13]
Median regression	0.0198	0.0311	0.1128***

*** significance at 1%, ** significance at 5%, * significance at 10%.
k = number of variables, n = number of observations.