such that

$$\hat{\beta} = \frac{p}{2}$$
$$\hat{\alpha} = 0$$

Further, effort is chosen so that

$$e^* = \frac{p}{2\delta}$$

The level of utility in equilibrium is strictly larger than the outside option (which was normalized to zero in this case).

6.4 The Basic Scheme when the Agent is Risk Averse

The presence of risk aversion changes the compensation schemes. In the baseline contract, the worker (or the agent) bears all the uncertainty associated with the luck component of output. The firm gets a fixed payment while the worker is obliged to observe fluctuations in her income. When the realization of the luck component is high, the compensation increases, while it falls when the realization of the luck component is low. As long as the worker is risk neutral, she does not care about income variability. This is no longer true in the case of risk aversion. When the worker is risk averse (and the firm risk neutral), the optimal distribution of risk calls for the firm to take up some of the risk. Indeed, as we will see, the optimal compensation implies a reduction in the bonus component and an increase in the fixed payment. In other words, the principal provides some insurance to the agent. Providing insurance, however, has an impact on effort and efficiency, which we know to be maximum under a pure bonus scheme. In other words, the optimal contract under risk aversion is the one that solves this *efficiency/insurance* trade-off.

6.4.1 THE AGENT PREFERENCES

General Agent Preferences. We now assume that the agent likes higher expected wage w^e but he dislikes income variability $Var(w)$. In this section we use a mean-variance utility function, whereby utility increases with the expected value w^e and decreases with the variance of the compensation and with the effort

$$U(w, e) = w^e - \lambda Var(w) - \delta \frac{e^2}{2}$$

The difference between risk aversion and risk neutrality on the agent's preferences is analysed next.

A risk-neutral agent: A risk-neutral agent is somebody who does not care about income variability, as was analysed in the first part of the chapter, so that for a risk-neutral guy $\lambda = 0$ or

$$\begin{aligned} U(w, e) &= w^e - \delta \frac{e^2}{2} \\ &= \alpha + \beta e - \delta \frac{e^2}{2} \end{aligned}$$

Risk-averse guy with mean variance preferences: A risk-averse guy with mean variance preferences is somebody who does care about income variability, so that his utility function is

$$U(w, e) = w^e - \lambda Var(w) - \delta \frac{e^2}{2}$$

The larger λ is, the more the worker is averse to income variability.

The linear compensation that we are analysing ($w = \alpha + \beta x$) has a variance that is simply given by $Var(w) = \beta^2 Var(x)$. Since the variance of the random variable x is constant and equal to v the variance of the compensation is $Var(w) = \beta^2 v$ so that the utility can be written as

$$U(w, e) = \alpha + \beta e - \lambda \beta^2 v - \delta \frac{e^2}{2}$$

6.4.2 THE PROBLEMS OF THE AGENT

We now solve the two problems of the agents under the case of risk aversion with mean variance preferences. We begin with the choice of effort under the general **bonus contract**: In the bonus contract $w = \alpha + \beta x$ so that $w^e = \alpha + \beta e$ and $Var(w) = \beta^2 v$ and the utility level is

$$Max_e : U(w, e) = \alpha + \beta e - \lambda \beta^2 v - \delta \frac{e^2}{2}$$

which implies that the optimal choice of effort is the condition (IC) above, or that

$$e^* = \frac{\beta}{\delta} \qquad \text{(IC: Incentive Compatibility Constraint)}$$

The choice of the optimal effort does not depend on the presence of risk aversion.

The second problem of the worker is whether participating in the contract is convenient, since the worker can enjoy an outside option which yields expected utility equal to u.

The utility of the agent is

$$U(w, e) = \alpha + \beta e^* - \lambda \beta^2 v - \delta \frac{e^{*2}}{2}$$

since $e^* = \frac{\beta}{\delta}$ it follows that participation is convenient if and only if

$$U(w(e^*), e^*)) \geq u$$

which implies that

$$\alpha + \frac{\beta^2}{\delta} - \frac{\beta^2}{2\delta} - \lambda \beta^2 v \geq u$$
$$\alpha + \frac{\beta^2}{2\delta} - \lambda \beta^2 v \geq u$$

so that

$$\alpha + \beta^2 [\frac{1 - \lambda v 2\delta}{2\delta}] \geq u \qquad \text{((P) Participation Constraint-Risk Aversion)}$$

The presence of risk aversion changes the participation constraint.

The presence of risk aversion makes participation in a bonus scheme a less attractive option to the worker. A bonus scheme is associated with variable wages, since the luck component of the output does affect the worker's utility. As one can see from the participation constraint required, the larger the risk aversion component v, for given β and α, the less likely is participation in the deal. We now move to the problem of the principal.

6.4.3 THE PROBLEM OF THE PRINCIPAL WITH A RISK-AVERSE AGENT: THE OPTIMAL BONUS SCHEME

The principal must now chooses $\tilde{\beta}$ and $\tilde{\alpha}$ where we indicate variables with a $\sim$ symbol to indicate that we are analysing the case of risk aversion.

Choice of $\tilde{\alpha}$. Let's first consider the optimal choice of $\tilde{\alpha}$ given $\tilde{\beta}$. For given choice of $\tilde{\beta}$ the principal will want to pay a salary so that the agent will choose to work. The agent chooses to work for this principal as long as he is just as well off as at his next-base opportunity. Thus, the smallest base pay $\tilde{\alpha}$ that the principal can offer, if she offers $\tilde{\beta}$, and still get the agent to work for her is

$$\tilde{\alpha} + \frac{\tilde{\beta}^2}{2\delta} - \lambda\tilde{\beta}^2 v = u$$

$$\tilde{\alpha} = u - \tilde{\beta}^2 \left[\frac{1 - \lambda v 2\delta}{2\delta}\right]$$

Choice of $\tilde{\beta}$. Let's study the profits of the principal when she offers $\tilde{\beta}$ and the agent is going to work by offering a level of $\tilde{\alpha}$ as indicated above. The expected profits of the agents are

$$E[\Pi] = pE[x] - w^e$$

$$= pe^* - \tilde{\alpha} - \tilde{\beta} e^*$$

$$\text{s.t. } e^* = \frac{\tilde{\beta}}{\delta} \text{ and } \tilde{\alpha} = u - \frac{\tilde{\beta}^2}{2\delta} + \lambda\tilde{\beta}^2 v$$

Substituting the two constraints in the objective functions one has

$$E[\Pi] = \frac{p\tilde{\beta}}{\delta} - u - \frac{\tilde{\beta}^2}{2\delta} - \lambda\tilde{\beta}^2 v$$

The principal will maximize profit with respect to $\tilde{\beta}$ so that the first-order condition solves

$$\frac{p}{\delta} - \frac{\tilde{\beta}}{\delta} - 2\lambda\tilde{\beta} v = 0$$

$$\tilde{\beta} = \frac{p}{1 + 2\lambda\delta v}$$

which implies that the optimal $\tilde{\beta}$ is less than the price (as long as $\lambda > 0$, $\upsilon > 0$ and $\delta > 0$).

For a risk averse guy, the optimal commission rate is never the franchising compensation scheme, i.e. $0 < \tilde{\beta} < p$

Summing up

With a risk-averse agent the scheme works as follows

$$\tilde{\beta} = \frac{p}{1 + 2\lambda\delta v}$$
$$\tilde{\alpha} = u - \tilde{\beta}^2 \left[\frac{1 - \lambda v 2\delta}{2\delta}\right]$$
$$\alpha = u - \frac{p^2}{(1 + 2\lambda\delta v)^2}\left[\frac{1 - \lambda v 2\delta}{2\delta}\right]$$

APPENDIX 6.1. WHAT IF THE PRINCIPAL AND THE RISK-AVERSE AGENT COULD CONTRACT ON EFFORT

Let's start by defining the surplus from the deal as the sum of the utility of each party, net of the respective outside option. This is identical to the difference between the revenues from the deal and the workers' cost of eliciting effort. The surplus is then

$$\begin{aligned} S &= (\Pi - 0) + (U - u) \\ &= pe - w + w - \frac{\delta e^2}{2} - u \\ &= pe - \frac{\delta e^2}{2} - u \end{aligned}$$

and let us assume that the two parties could contract directly on effort, rather than relying on the output x. Let's see what the effort choice is that maximizes the surplus from the job. In other words, the optimal effort is the one that makes the marginal surplus zero ($i.e. \frac{\partial S}{\partial e} = 0$). We call the optimal level of effort e^0 and it is easy to see that

$$p - \delta e^o = 0$$

from which it follows that

$$e^o = \frac{p}{\delta}$$

The level of effort that maximizes the surplus is $e^0 = p/\delta$.

So that the maximum value of the surplus is (evaluating S at e^o)

$$S(e^o) = \frac{p^2}{\delta} - \frac{\delta p^2}{\delta^2 2} = \frac{p^2}{2\delta}$$

and the job is efficient as long as

$$S(e^o) > 0$$
$$\frac{p^2}{2\delta} > u$$