

## Problem Set 8

### Subjective Evaluation

#### Main Points

##### Gaming

- Often, the real outcome that the principal cares about cannot be verified and therefore contracts based on this outcome cannot be enforced in courts.
- When the agent can manipulate the verifiable performance measures, it is optimal not to tie too much of the agent's pay to these measures. This in general causes efficiency loss.
- This efficiency loss can be eliminated if the principal can design a contract based on the unverifiable outcome that she cares about.
- The contract based on the unverifiable outcomes must be self-enforcing. This will in general be the case when the principal cares about future interactions with the agent(s) or when the verifiable performance measures are very corrupt.

##### Multilayer Organizations

- In some relationships, such as those in the multilayer organizations, the principal is not the residual claimant.
- Even though in this case the principal does not directly benefit for misreporting on the agent's performance that cannot be verified, the evaluation may still be biased for other reasons.
- These reasons include: costly information gathering, cognitive limitation, negative personal consequences, and favoritism.
- Biased rating can be prevented using several methods, such as: punishing biased rating, forced ratings, training, and bureaucratic rules.

#### Main Concepts

Objective performance measures; Subjective performance measures; Gaming; Self-enforcing contracts; Trigger strategy; Discounting; 'Halo' effect; Rent seeking; Forced rating; 'Bureaucratic' rules.

#### Problems

- (1) Long-term success of the Republican Party in the U.S. depends in part on Sarah Palin's promotional effort ( $e$ ). Specifically, the probability that the Party will win is given by  $\Pr(q=1|e)=e$ , where  $q$  is 1 if the Party wins and 0 otherwise. Sarah's compensation is a pay for performance contract  $w=a+by$ , where  $y$  is the percent of U.S. citizens who claim they would vote Republican if the election was held today. Sarah is risk-neutral and her cost of effort is given by  $0.5e^2$ . The Party knows that  $y$  is proportional to  $q$  according to  $y=kq$ , where  $k$  has a mean of 1 and a variance of 3. On the other hand, Sarah knows and can influence the value of  $k$ . Find the optimal value of  $b$  and show that it in general induces an inefficiently low promotional effort.

- (2) Employee's contribution to the firm is given by  $q=e+u$ , where  $e$  is employee's effort and  $u$  is a random variable with a mean of zero. The cost of effort function is  $e^2/6$ . Both the employee and employer are risk neutral and have an outside option of zero. Compare the value of the expected social surplus when the employer uses  $q$  as a performance measure and when the employer uses  $y=kq$  as a performance measure, where  $k$  is a parameter that the employee can manipulate. The employee knows that the actual value of  $k$  is 1, while the employer only knows that  $k$  has a mean of 1 and a variance of  $\theta=1$ .
- (3) A cardiovascular surgeon can perform  $q$  surgeries per day according to  $q=4e+u$ , where  $e$  is physician's effort and  $u$  is a random variable with a mean of 0 and a variance of 2. The physician's disutility of effort is  $c(e)=0.5e^2$  and his outside option is \$0. The physician is risk averse, with the coefficient of absolute risk aversion equal to 3. The Toronto General Hospital (TGH) employs the surgeon and is risk-neutral. How much would the TGH be willing to pay to obtain a subjective performance measure that would completely eliminate uncertainty?
- (4) The quality of General Motors' (GM) cars depends on the assembly line worker's effort  $e$  according to  $q=10e$ . The cost of effort to the worker is  $e^2$  and his outside option is  $R=10$ . The GM management can observe worker's effort but it cannot verify it to a third party. The payment contract for the worker is  $w$  if his effort level is at or above the efficient effort level and 0 otherwise. Show that the GM management has an incentive to misreport the actual worker's effort if it expects the company to go bankrupt after one period. Assume that if the GM misreports the worker's actual effort, this becomes public information and no worker in the future wants to work for the GM.
- (5) Consider the model described in question 4. Suppose that the U.S. government provides a large subsidy to GM, which saves it from bankruptcy. The GM now expects to be in business forever. The GM management realizes that if it misreports the actual effort of the worker, it will not be able to hire any workers in the future. If the GM management discounts the future profits at rate  $\beta=0.9$ , will it develop its reputation as a good firm or will it shut down after one period?

### Suggested Solutions

(These solutions are intended to be accurate and as complete as possible. Please report any remaining errors to [jasmin.kantarevic@oma.org](mailto:jasmin.kantarevic@oma.org) )

(1) Sarah's expected payoff is  $E[U]=E[w-c(e)]=E[a+by-0.5e^2]=E[a+bkq-0.5e^2]$ . After Sarah observes  $k$ , she will choose  $e$  to maximize  $E[U]=a+bke-0.5e^2$ . The first-order condition for  $e$  is  $bk=e$ . This is the incentive compatibility constraint (IC). The participation constraint (PC), before Sarah observes  $k$ , is  $E[U]=E[a+bkq-0.5e^2]=R$ , or  $a=R-E[bkq-0.5e^2]$ . The Party's expected payoff is  $E[V]=E[q-w]=E[q-bkq-a]$ . Substituting for PC and then IC yields  $E[V]=E[e-0.5e^2-R]=E[bk]-0.5E[(bk)^2]-R=b-0.5b^2(1+\theta)$ . Substituting for  $\theta=3$ , this yields  $E[V]=b-2b^2$ . The first-order condition for  $b$  is then  $1-4b=0$ . Therefore,  $b=0.25$ . From IC, this implies that  $e=bk=0.25k$ . On the other hand, if the information was symmetric, the efficient level of effort would maximize  $E[e-0.5e^2-R]$ , which implies that  $e^*=1$ . Therefore, for  $k>4$ ,  $e>e^*=1$  and for  $k<4$ ,  $e<e^*=1$ . Given that  $E[k]=1$ , on average  $e=0.25<e^*=1$  and Sarah will in general provide inefficiently low promotional effort.

(2) When the employer uses  $q$  as a performance measure, he can induce the efficient level of effort by using a piece rate contract with  $b=1$ . This level of effort is given by equating the expected marginal benefit and marginal cost of effort. In this question, this implies that  $1=e^*/3$ , so  $e^*=3$ . The value of social surplus is then  $E[q(e^*)-c(e^*)]=3-3^2/6=1.5$ . When the employer uses  $y$  as a performance measure, the employee maximizes  $a+bE[y]-c(e)=a+bke-e^2/6$ . The IC condition is then  $bk-e/3=0$  or  $e=3bk$ . The employer maximizes his expected payoff subject to IC and PC. The employer's expected payoff is  $E[e-c(e)]=3bE[k]-E[(3bk)^2]/6=3b-1.5b^2E[k^2]=3b-1.5b^2(1+\theta)=3b-3b^2$ . The optimal value of  $b$  is then  $0.5$ . Given  $b=0.5$ , the employee will choose  $e=3bk=1.5$ . The social surplus when the employer uses  $y$  is then  $1.5-1.5^2/6=1.125$ .

(3) With no uncertainty, the hospital can implement the efficient level of effort by equating the marginal benefit of effort (4) and the marginal cost of effort ( $e$ ). Therefore,  $e^*=4$ . Also, given no uncertainty, the hospital can use a simple salary contract that satisfies the surgeon's participation constraint:  $w=R+c(e^*)=0+0.5e^{*2}=0.5(4^2)=8$ . The social surplus is then  $q(e^*)-c(e^*)=4(4)-8=8$ . When there is uncertainty, the hospital can use a piece-rate contract  $a+bq$ . To determine the value of this contract, we can proceed in three steps. First, the expected payoff for the surgeon is  $E[w]-R-c(e)=a+4be-0.5rb^2\theta-0.5e^2$ . Therefore, the first-order condition for  $e$  is  $4b-e=0$ . This is the IC. Second, the PC is  $E[w]=R+c(e)+RP$ . Lastly, we can substitute for IC and PC in the hospital's expected payoff to get  $E[q-w]=4e-0.5rb^2\theta-0.5e^2-R=16b-0.5rb^2\theta-8b^2$ . The first-order condition for  $b$  is then  $16-rb\theta-16b=0$ , from which it follows that  $b=16/(16+r\theta)=16/(16+3(2))=16/22\approx 0.7$ . From the participation constraint, we then have that  $a+4be-0.5rb^2\theta-0.5e^2=0$ , which yields  $a=0.5(3)(0.7^2)(2)+8(0.7^2)-4(0.7)(4*0.7)\approx 5.8$ . Therefore, the physician's total expected compensation is  $E[w]=a+4be\approx 13.9$ . The expected profit is then  $E[q]-E[w]=4(4)(0.7)-13.9<0$  and the hospital would not employ the physician and its profit would be 0. Therefore, the maximum the hospital would be willing to pay to remove all uncertainty is 8.

(4) The efficient level of effort is given by the  $MB(e^*)=MC(e^*)$  condition, which in this case yields  $e^*=5$ . The salary level is then determined from the participation constraint as  $w=R+c(e)=10+5^2=35$ . The GM's profit for one period if it reports  $e<5$  is  $10(5)-0=50$  and its profit if it reports  $e=5$  is  $10(5)-35=15$ . Therefore, if the GM expects to go bankrupt after one period, it will have an incentive to under-report the worker's true effort.

(5) If the GM management misreports the worker's actual effort, it will earn 50 this period and then it will earn zero forever since no worker will want to work for it. If the GM management chooses to report truthfully, its profit will be 15 forever, which has the present value of  $15/(1-0.9)=150$ . Therefore, it is in the GM management's best interest to develop reputation as a good firm.