Problem Set 6

Multitasking

Main Points

- Multitasking and power of incentives: In general, contracts based on multiple tasks should tie less of the agent's pay on performance because of the increased risk that the agent must take.
- Equal compensation principle: In general, the agent supplies inefficiently low effort for tasks that are not rewarded.

Main Concepts

Multitasking; equal compensation principle.

Problems

(1) The hockey player's contribution to the team is given by $q=e_1+e_2$, where e_1 represents effort expended on scoring goals and e_2 represents effort expended on assisting others to score goals. The team manager can't observe either e_1 or e_2. However, he can observe the number of goals scored (y_1) and the number of assists (y_2). The number of goals scored is given by $y_1=e_1+u_1$, where u_1 is a random variable with a mean of zero and a variance of 1. Similarly, the number of assists is given by $y_2=e_2+u_2$, where u_2 is a random variable with a mean of zero and a variance of 1. Assume that u_1 and u_2 are not correlated. The player is risk averse with a coefficient of absolute risk aversion of 2 and a cost of effort function $c(e_1,e_2)=0.5(e_1+e_2)^2$. The contract specifies that the player's total compensation equals $w=a+b_1y_1+b_2y_2$. What is the optimal compensation weight on goals scored (b_1) and assists (b_2)?

(2) Your learning in this course is given by $q=e_1+ke_2$, where e_1 represents effort expended on attending classes, e_2 represents effort expended on doing assignment problems, and $k>0$ is a constant that reflects the relative contribution on e_2 to your learning. Your professor cannot observe either e_1 or e_2. However, he can observe two signals, y_1 and y_2¸that are related to your effort on each task. Specifically, $y_1=e_1+u_1$, with $E[u_1]=0$ and $Var[u_1]=\theta_1$, and $y_2=e_2+u_2$, with $E[u_2]=0$ and $Var[u_2]=\theta_2$. You are risk neutral and your cost of effort is given by $c(e_1,e_2)=0.5(e_1^2+e_2^2)$. If your grade in this course is given by $w=a+b_1y_1+b_2y_2$, show that the professor who is risk neutral will set $b_1>b_2$ only if $k<1$ (i.e. attending classes will be rewarded relatively more only if it contributes to your learning more than doing assignment problems).

(3) The Ministry of Health is interested in both quantity and quality aspects of physician practice. Specifically, the outcome is $q= e_1+e_2$, where e_1 represents effort expended on seeing more patients and e_2 represents effort expended on providing better quality service to each patient. The Ministry cannot observe either e_1 or e_2. However, it can observe two signals, y_1 and y_2¸ that are related to the physician effort on each task. Specifically,

$y_1=e_1+u_1$, with $E[u_1]=0$ and $Var[u_1]=\theta_1$, and $y_2=e_2+u_2$, with $E[u_2]=0$ and $Var[u_2]=\theta_2$. The physician is risk neutral and his cost of effort is given by $c(e_1,e_2)=0.5(e_1^2+ke_2^2)$. The contract specifies the physician pay as $w=a+b_1y_1+b_2y_2$. Find the optimal weights b_1 and b_2 and the effort levels that these weights induce.

(4) Auto insurance agents often are expected to provide on-going customer service after a policy is sold (answering questions about the policy, providing assistance in filing claims, and so on). Explain why some auto insurance companies separate the tasks of selling and customer service. Which of the two jobs should be tied more to measurable performance?

(5) Jane Hannaway (1992) proposes to separate the jobs of teaching basic and higher skills. How does she respond to the following objections to her argument?

- It is inefficient to separate jobs because they are complementary tasks;
- Separating the job may introduce co-ordination problem between teachers.
- Teachers may not be motivated by economic incentives.

Suggested Solutions

(These solutions are intended to be accurate and as complete as possible. Please report any remaining errors to jasmin.kantarevic@oma.org).

(1) The player's expected payoff is $E[U]=E[w]-RP^A-c(e)=a+b_1e_1+b_2e_2-0.5r(b_1^2\theta_1+b_2^2\theta_2)-0.5(e_1+e_2)^2$. To find the incentive compatibility constraint (ICC), note that the first-order conditions for e_1 and e_2 are $b_1=e_1+e_2$ and $b_2=e_1+e_2$, respectively. This implies that $b_1=b_2$. Let this common value be denoted by b. Also, let e_1+e_2 be denoted by e. Therefore, we can write the IC as b=e. Also, the participation constraint (PC) is $E[w]-RP^A-c(e)=R$. Using the IC and PC, we can write the manager's expected payoff as $E[V]=E[q]-E[w]=e-0.5rb^2(\theta_1+\theta_2)-0.5e^2-R= b-0.5rb^2(\theta_1+\theta_2)-0.5b^2-R$. The manager maximizes his expected payoff by choosing the value of b. The first-order condition for b is $1-rb(\theta_1+\theta_2)-b=0$, from which it follows that $b=1/[1+r(\theta_1+\theta_2)]$. Substituting for the values of $r=2$ and $\theta_1=1=\theta_2$ given in the problem, this implies that $b=1/5=0.2=b_1=b_2$.

(2) Your expected payoff is $E[U]=E[w]-c(e)= a+b_1e_1+b_2e_2-0.5(e_1^2+e_2^2)$. (Note that there is no risk premium since you are risk neutral.) You will choose your effort level to maximize your expected payoff, so the first-order conditions for e_1 and e_2 are $b_1=e_1$ and $b_2=e_2$, respectively. This is the IC. The PC is simply that $E[w]=R+c(e_1,e_2)$. Substituting for the IC and PC in the professor's expected payoff, we have $E[V]=E[q]-E[w]= e_1+ke_2-0.5(e_1^2+e_2^2)-R = b_1+kb_2-0.5(b_1^2+b_2^2)-R$. The first-order conditions for b_1 and b_2 are $1-b_1=0$ and $k-b_2=0$, respectively, which implies that $b_1=1$ and $b_2=k$. Therefore, $b_1>b_2$ if and only if $k<1$.

(3) The physician's expected payoff is $E[U]=E[w]-c(e)= a+b_1e_1+b_2e_2-0.5(e_1^2+ke_2^2)$. (Note that there is no risk premium since the physician is risk neutral.) The physician will choose his effort levels to maximize his expected payoff, so the first-order conditions for e_1 and e_2 are $b_1=e_1$ and $b_2=ke_2$, respectively. This is the IC constraint. The PC is simply that $E[w]=R+c(e_1,e_2)$. Substituting for the IC and PC in the Ministry's expected payoff, we have $E[V]=E[q]-E[w]= e_1+e_2-0.5(e_1^2+ke_2^2)-R = b_1+b_2/k-0.5(b_1^2+k(b_2/k)^2)-R$. The first-order conditions for b_1 and b_2 are $1-b_1=0$ and $1/k-b_2/k=0$, respectively. Therefore, $b_1=1$ and $b_2=1$ and from the ICC we have $e_1=1$ and $e_2=1/k$.

(4) Sales and customer service are both valuable to the company, but the quality of service is much harder to measure. If both tasks are valued, and service is hard to measure, then it is optimal not to provide any incentives to either task (i.e. salary). This is inefficient since the agent then may not try hard on either task. The firm can do better by separating the tasks and paying salesmen based on the value of sales and paying customer service representatives a fixed salary.

(5) See Hannaway (1992).