Problem Set 5

Multiple Signals

Main Points

- Informativeness Principle: An additional signal may reduce risk and improve incentives in the principal-agent model with hidden action if the signal is informative about the agent's performance. In this case, the compensation should vary positively with the signal if the signal and performance are negatively related, and vice versa.

Main Concepts

Informativeness principle.

Problems

(1) Family Guy is an animated television sitcom created by Seth MacFarlane that airs on Fox. The Fox management is considering changing the compensation structure for MacFarlane from w=a+0.2q to w=a+0.2q+cy, where q is the number of Fox viewers and y is the number of CNN viewers. The expected number of Fox viewers is e and its variance is 1, while the expected number of CNN viewers is 0 and its variance is 0.5. In addition, the number of Fox and CNN viewers is negatively correlated, with the covariance of -0.5. Assume that MacFarlane is risk averse with the coefficient of absolute risk aversion equal to 3. Find the risk premium for MacFarlane if the Fox management optimally uses signal y. Compare this risk premium to the risk premium when the Fox management does not use signal y.

(2) University professors are currently compensated according to a pay for performance contract w=a+bq, where q is the number of research articles that the professor publishes in economic journals. The Board of Directors is considering introducing an additional performance measure in the professors' contract, such as students' evaluation scores. Explain how the Board should use this additional measure if the professors are risk neutral.

(3) Suppose that the proposed contract is w=a+b(q+cy), where q is agent's performance and y is another signal of agent's performance. Suppose further that E[q]=E[y]=0, Var[q]=θ, Var[y]=1, and Cov[q,y]= ρ. What is the value of c that minimizes the variance of the agent's pay?

(4) Wolgang Puck, a celebrity chef from California, wishes to hire a manager for his new restaurant. Mr. Puck proposes to pay w=a+bq+cy, where q is the number of customers in his restaurant and y is the number of customers in a restaurant of similar quality. Specifically, Mr. Puck knows that E[q]=e, Var[q]=2, E[y]=0, Var(y)=1, and cov(q,y)=0.9. In addition, Mr. Puck knows that the manager's cost of effort is $0.5e^2$ and his coefficient of risk aversion is r=2. Show that the expected number of customers in

Mr. Puck's restaurant is greater if the contract is based on both q and y rather than on q alone.

(5) Suppose that the agent's output is q=e+u, where e is effort and u is a random variable. Suppose further that u=βy+v, where y is an additional signal and E[v]=0. Show that if the principal pays the agent based on the expected effort given signal y, the contract has the form q=a+bq-bβy.

(6) Suppose that the agents performance q and signal y are perfectly correlated (i.e. knowing one perfectly predicts the other). Formally, this can be expressed as saying that the coefficient of determination, R^2, defined as $cov^2(q,y)/[Var(q)Var(y)]$, is equal to 1. Consider the case where the proposed contract is w=a+b(q+cy), where E[q]=E[y]=0, Var[q]=θ, Var[y]=1, and Cov[q,y]= ρ. Show that the optimal choice of using the signal, c*, in this case can completely eliminate variation in w.

(7) A researcher wished to test whether contracts based on multiple performance signals improve workers' productivity relative to contracts based on only one signal. To that purpose, the researcher collected data on the type of contracts and the workers' productivity and estimated a regression model using the least-squares method. The estimation results are shown in the table below:

	Dependent variable = Productivity	
Independent variables	Coefficient	Standard Error
Constant	4.1	2.4
Type of contract (=1 if multiple signals, =0 if only one signal)	0.6	0.2

a. Define the hypothesis to be tested and how it relates to theory.
b. Define treatment and the treatment and control groups for this problem.
c. Identify assumptions needed to strengthen inference about the causal impact of treatment.
d. Interpret the results (e.g. the statistical and economic significance of estimates; the relation between the results and specific theoretical predictions).

(8) Suggested Solutions

(These solutions are intended to be accurate and as complete as possible. Please report any remaining errors to jasmin.kantarevic@oma.org .)

(1) The risk premium is 0.5rVar[w], where r is the coefficient of absolute risk aversion. When the company uses signal y, we have that $Var[w] = Var[a+bq+cy] = b^2Var(q)+c^2Var(y)+2bccov(q,y)$. Substituting for the values given in the question, we have that $Var[w]=(0.2)^2(1)+c^2(0.5)+2(0.2)c(-0.5)=0.04+0.5c^2-0.2c$. The optimal use of signal y is such to minimize this variance. The first-order condition with respect to c is then c-0.2=0, c*=0.2. Then, the risk premium is $0.5rVar[w]=0.5(3)[0.04+0.5(0.2)^2-0.2(0.2)]=0.03$. On the other hand, when the company does not use signal y, we have that $Var[w]=Var[a+bq]=b^2Var(q)=(0.2)^2(1)=0.04$ and the risk premium is then 0.5rVar[w]=0.5(3)(0.04)=0.06. Therefore, the risk premium is higher when the company does not use the signal, i.e. using the signal reduces uncertainty in the agent's payoff.

(2) The potential benefit of using additional signals is to reduce uncertainty in the professor's pay and therefore also improve incentives by increasing the value of b. However, when the professors are risk neutral, they are not concerned with risk and the principal can set b equal to its optimal value of 1. In other words, there is no benefit of using additional signals of performance when the agent is not concerned with uncertainty.

(3) The variance in this problem is $Var[w]=Var[a+b(q+cy)]=b^2(\theta+c^2+2c\rho)$. The first-order condition for c is then $b^2(2c+2\rho)=0$, which yields $c^* = -\rho$.

(4) Consider first the contract that does not include y, i.e. w=a+bq. We have that $E[U]=E[w]-0.5rVar[w]-c(e)=a+be-0.5rb^2\theta-0.5e^2$. Therefore, the incentive compatibility constraint (IC) yields $\partial E[U]/\partial e=b-e=0$, or b=e. In addition, the participation constraint (PC) gives us E[U]=R+c(e)+0.5rVar[w]. Substituting these two constraints in the principal's expected payoff, we have $E[V]=E[q]-E[w]=b-0.5rb^2\theta-0.5b^2-R$. The first-order condition for b is then $1-rb\theta-b=0$, which gives $b=1/(1+r\theta)$. Substituting for r and θ from the problem, we then have that b=1/(1+2(2))=1/5=$\boxed{0.2}$. This is also the expected number of customers since from the IC, b=e.

Consider next the contract that includes y, i.e. w=a+bq+cy. The agent's expected payoff now becomes $E[U]=E[w]-0.5rVar[w]-c(e)=a+be-0.5r(b^2\theta+c^2+2bc\rho)-0.5e^2$. Therefore, from the first-order condition for b, the IC constraint is still b=e. Also, we have from the PC that E[w]=R+c(e)+0.5rVar[w]. Substituting these two constraints in the principal's expected payoff, we have that $E[V]=E[q]-E[w]=b-0.5r(b^2\theta+c^2+2bc\rho)-0.5b^2-R$. We now need the first-order conditions for both b and c. For b, the FOC is $1-rb\theta-rc\rho-b=0$ and for c we have $rc+rb\rho=0$. Therefore, $c = -b\rho$. Substituting in the FOC for b we then have that $1-rb\theta-r(-b\rho)\rho-b=0$, which yields $b=1/(1+r\theta-r\rho^2)$. Substituting for values of r, θ, and ρ from the problem, we have that $b=1/(1+2(2)-2(0.9^2))\cong\boxed{0.3}$. This is also the expected number of customers, since from the IC we have that b=e. Notice that this is higher than when the signal is not used. Lastly, we can also find c from the first order condition $c=-b\rho=-0.3(0.9)=-0.27$. Therefore, the optimal contract is w=a+0.3q-0.27s.

(5) The problem is that the principal observes only q and cannot separate the effect of e and u on q. However, the principal can learn something from observing y. In particular, $E[u|y]=\beta y$. Therefore, the principal can conclude after observing y that $E[e|y]=q-E[u|y]=q-\beta y$. If the agent is paid based on the expected output, then the pay is $w=a+bE[e|y]=a+b(q-\beta y)=a+bq-b\beta y$.

(6) We have, as in Question 3, that $Var[w]=Var[a+b(q+cy)]=b^2(\theta+c^2+2c\rho)$. The first-order condition for c is then $b^2(2c+2\rho)=0$, which yields $c^*=-\rho$. Using this, $Var[w]= =b^2(\theta+\rho^2-2\rho^2)=b^2(\theta-\rho^2)$. Now, if q and y are perfectly correlated, $R^2=1$. Using the definition of $R^2=cov^2(q,y)/[Var(q)Var(y)]= \rho^2/[\theta(1)]= \rho^2/\theta=1$, or $\rho^2=\theta$. Using this, we have that $Var[w]=b^2(\theta-\rho^2)=0$. Therefore, a contract $a+b(q-\rho y)$ can completely eliminate uncertainty when q and y are perfectly correlated.

(7) (**a**) The hypothesis to be tested is whether contracts based on multiple performance signals improve workers' productivity relative to contracts based on only one signal. This hypothesis is related to the theory of optimal incentives in which the principal cannot observe or verify the agent's actions, the outcome that depends in part on the agent's action is uncertain, and the agent is risk averse. In this case, using a signal that is informative about the agent's effort can reduce risk in the agent's pay and more of the agent's pay can be tied to performance, which may improve incentives. However, using signals that are not informative about the agent's action will in general have no impact on incentives. Therefore, the hypothesis tests both whether there is an impact of using multiple signals and whether in practice the firms use signals that are informative. (**b**) The treatment in this problem is whether the worker is compensated using a contract based on multiple signals. The treatment group is the group of workers compensated using this type of contract, while the control group is the group of workers compensated using a contract based on only one signal. (**c**) The comparison of productivity between treatment and control worker will be informative about the causal impact of treatment only if the two groups are similar in all other relevant aspects, such as skill level, experience, type of industry, etc. (**d**) The results indicate that using contracts with multiple signals have a positive impact (0.6) on workers' productivity. Moreover, this impact is statistically significant ($t=0.6/0.2 > 2$). The impact is also economically significant, representing about 15% improvement in productivity relative to contract that use only one signal (0.6/4.1). Therefore, this result is consistent with the prediction that using contracts based on multiple signals may improve workers' productivity.