Problem Set 4

Risk-Neutral Agent

Main Points

- Risk Neutral Agent: When the agent is risk neutral, the moral hazard is not a problem because the principal can sell the 'job' to the agent and let the agent keep all output.
- Treatment and Selection Effects: Testing whether one payment method improves productivity compared to another method faces the challenge of separating the observed difference in productivity into the effect that can be attributed solely to the payment method (treatment effect) and the effect of all other differences between workers in different payment models (selection effect).
- Randomized Experiments: Randomization of workers into different payment methods ensures that the workers are similar in all other aspects. The treatment effect can then be obtained by simply comparing the average difference in productivity between workers in the different payment models.

Main Concepts

Regression model; Least-Squares Method; Statistical and economic significance of regression coefficients; Treatment effect; Selection effect; Identification problem; Confounding factors; Randomized experiment; External validity.

Problems

(1) The Ministry of Health wants to employ a physician to treat elderly patients in a long-term care home. On average, the physician can treat 10 patients per day. The physician disutility of work is $0.5e^2$ and his outside option is \$25, while the Ministry's outside option is \$0. Is it efficient that the Ministry employs the physician? If so, what contract of the form w=a+bq should the Ministry offer to the physician if it cannot verify how many days the physician works? Assume that both the physician and the Ministry are risk neutral.

(2) The quality of students' learning often depends not only on teacher's effort, but also on her ability to transfer knowledge. Specifically, suppose that E[q]=e+n, where q is the quality of students' learning, e is teacher's effort, and n is teacher's ability to transfer knowledge. Assume also that the teacher is risk neutral and that her disutility of effort is $0.5e^2$. Explain how differences in ability across teachers affect the design of the optimal piece rate contract when the teacher's effort and ability cannot be observed.

(3) A sales rep is paid a fixed salary of \$20 per day if she sells 5 shirts or less and \$10 per shirt if she sells more than 5 shirts. What is the equivalent contract of the form w=a+bq?

(4) The class size may affect learning outcomes. Specifically, the learning outcome for children in classes with 20 students or less is given by 10+u, while the learning outcome

for children in classes with more than 20 students is given by 5+u, where u is the individual student's ability. You learned that the average ability of students in classes with 20 students or less is 4 and the average ability of students in classes with over 20 students is 2. What is the magnitude of treatment and selection effects of being in a class with 20 or less students in the class?

(5) The Ministry of Health conducted a randomized experiment to measure whether fee-for-service physicians (i.e. physicians paid according to a piece rate system) treat more patients than salaried physicians. Prior to the experiment, all physicians were salaried physicians. Physicians were randomly assigned to either the fee-for-service pool of physicians or the salary pool of physicians. The following table shows the results:

	Patients seen per day before the experiment	Patients seen per day after the experiment
Fee-for-Service	20	25
Salary	20	20

a) Does the data support the claim that the randomization assigns similar physicians to the treatment (fee-for-service) and control (salary) groups?
b) What do these results say about the productivity difference between the fee-for-service and salary physicians?

(6) Explain why, in his tree-planting experiment, Bruce Shearer (2002) assigned the same supervisor to both tree planters that were paid hourly wage and tree planters that were paid based on how many trees they plant.

(7) Allen-Edmonds Shoe Company is a manufacturer of high-priced shoes. For years, it paid its factory employees based on individual output through a piece rate system. In 1990, following the advice of quality gurus, the company abandoned the piece-rate system and started paying employees fixed hourly wages. After the policy change, the average productivity of employees decreased by 10 percent. Discuss whether this result is consistent with economic theory and empirical evidence from Shearer (2002).

(8) Some parents tend to be late in picking their children from day care centers. As a solution, consider implementing a fine for the tardy parents (the fine system) and using a randomized experiment to test whether this solution would work.

a. Clearly define the following terms as they apply to this experiment: unit of observation-outcome – treatment – treatment group – control group.
b. Clearly state the research question in a cause-effect form.
c. Discuss why a comparison of childcare centers that choose to adopt the fine system and childcare centers that choose not to adopt the fine system may not represent the causal effect of implementing the fine system.
d. Explain how you would design a randomized experiment for this purpose and why it may help you uncover the causal relationship of interest.
e. Discuss what other confounding factors you would have to control for in your experiment.

(9) A researcher has collected data on the workers' productivity (Y), their payment method (T=0 if salary, T=1 is piece rate), and their years of experience (X). Using these data, she has estimated the following regression model:

Variable	Coefficient	Standard Error
Constant	20	4.0
Compensation Type	4	1.0
Experience	2	0.5

a. Using the estimated coefficients, write down the equation describing the relationship between Y and each of T and X.
b. What is the expected productivity of a salary worker with no experience?
c. What is the expected productivity of a piece rate worker with 10 years of experience?
d. Are piece rate workers more productive than salary workers?
e. How does experience affect productivity?

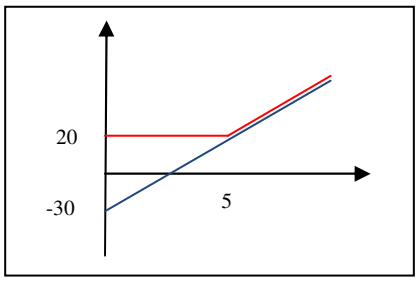
Suggested Solutions

(These solutions are intended to be accurate and as complete as possible. Please report any remaining errors to jasmin.kantarevic@oma.org .)

(1) It is efficient for the Ministry to hire the physician if the value of the relationship evaluated at the optimal level of effort exceeds the sum of the outside options for the Ministry and the physician. That is, $E[q]-c(e)\geq R+S$, or $10e^*-0.5e^{*2}\geq 25$. The optimal level of effort e* satisfies $E[q'(e^*)]=c'(e^*)$, or $10=e^*$. This implies that $10e^*-0.5e^{*2}=50$, which is greater than 25, and therefore it is optimal that the Ministry hires the physician. To find the optimal contract [a*,b*], we start with the Ministry's expected payoff $E[V]=E[q]-a-bE[q]=10e(1-b)-a$. In the second step, we use the physician's participation constraint (PC) to substitute for a. The PC is $E[U]=a+bE[q]-c(e)=R$, or $a+b10e-0.5e^2=25$, or $a=25+0.5e^2-10be$. Substituting for a in the Ministry's expected payoff yields $E[V]=10e(1-b)-25-0.5e^2+10be=10e-0.5e^2-25$. In the third step, we substitute for e from the incentive compatibility constraint (IC). The IC constraint requires that e be such to maximize the physician's expected payoff given the contract [a, b] : e maximizes $E[U]=a+b10e-0.5e^2$. The first-order condition for e yields e=10b. Using this expression for e, we have that $E[V]=10e-0.5e^2-25=100b-50b^2-25$. The first-order condition for b yields b*=1. From the IC constraint, we then have that e*=10b*=10. From the PC, we then have that $a^*= 25+0.5e^{*2}-10b^*e^*=-25$. Therefore, the Ministry should offer [a*,b*]=[-$25,1]. This piece rate contract will achieve the optimal level of effort e* of 10.

(2) Given any linear contract of the form w=a+bq, the teacher's expected payoff is $E[U]=a+bE[q]-c(e)=a+b(e+n)-0.5e^2$. When the teacher is risk neutral, b*=1. Further, from the incentive compatitibility constraint, we have that e*=b*=1. Therefore, E[U]=a+1+n-0.5=a+n-0.5. From the participation constraint, we then have that a*=0.5+R-n. Now, this contract will be acceptable to all teachers with ability greater than 0.5+R-a* and not acceptable to all other teachers. These results can be summarized as follows: teachers with higher ability will be more likely to accept the contract, but all teachers who accept the contract will provide the efficient level of effort.

(3) The payment is $20 for q≤5 and $10q for q>5. We want to express this in the form of w=a+bq. First, we know that b=10. Second, we also know that for q=5, w=20=a+10(5), from which it follows that a=-30. Therefore, an equivalent contract has the form w=-30+10q. This can be illustrated in the following graph.



(4) The treatment effect is the difference in learning outcomes between the two types of classes, holding the student ability constant: 10-5=5. The selection effect is the difference between the observed difference in the learning outcomes and the treatment effect. The observed learning outcome is 10+4=14 in the class with 20 students or less and 5+2=7 in the class with more than 20 students. Therefore, the observed difference is 7, of which 5 is the treatment effect and the remaining 2 is the selection effect.

(5) (a) Yes, since the number of patients seen per day is identical for the two groups prior to the experiment (i.e. 20). (b) The results indicate that the fee-for-service physicians treat 25 patients per day. This is 5 patients more than the control group of salary physicians. Therefore, the treatment effect of piece rate is 25-20=5 patients, or 5/20=25 percent.

(6) Shearer was concerned that, even after randomization, his treatment (piece-rate planters) and control (wage planters) groups may be systematically different. For example, suppose that strict supervisors were assigned to the wage planters and lenient supervisors to the piece-rate planters. In this case, the number of trees planted by the wage planters may be higher than the number of trees planted by the piece-rate planters if they were both assigned the same type of supervisors. This difference in the number of trees would have nothing to do with how planters are paid. This is an example of how correlation between two variables (how workers are paid and their productivity) may not represent a cause-effect relationship because a third variable (type of supervisor) may affect both variables.

(7) This result is consistent with the economic theory of piece rates when the agent's effort cannot be observed and both parties are risk-neutral. Specifically, the piece rate pay induces the efficient level of effort by effectively renting the job to the worker and letting the worker keep the value of his output. When effort is not observed, the salary contract will elicit less than the optimal level of effort. The direction of the change is consistent with the empirical evidence in Shearer (2002) who showed that the tree planters in B.C. who are paid based on the number of planted trees are more productive than the planters paid by the hourly wage. However, Shearer estimates that the difference in productivity is about 20 percent, which is double what is observed for the case of Allen-Edmonds Shoe Company. The difference in the magnitude can perhaps be explained by the selection effect. For example, some productive workers may not have left the company after the salary system was implemented for reasons other than the change in how the workers are paid.

(8)

a. Outcome could be defined, for example, as the number of days per week that the parents are late. The unit of observation could be defined as the day care centre. The treatment is the presence of the fine system. The treatment group is the group of daycare centers with the fine system in place, while the control group is the group of daycare centers with no fine system in place.
b. "Does the fine system for tardy parents (cause) reduce the number of days per week that parents are late in picking their children from daycare centers (effect)?"
c. The observed difference in the outcome between the two types of daycare centers may reflect not only the impact of the fine system but also other differences that may affect the outcome, such as the size of the daycare centre, its location, and differences in management.
d. You can start with a randomly selected sample of daycare centers, all of which have no fine system in place. Using a random device, you can then assign half of these centers to the fine system while the other half continues with no fine system. This may help uncover the causal effect because randomization would help reduce differences between the centers that may affect the outcome.
e. Few examples include the size of the centre, its location, the qualification of supervisors, and the average income and education of parents. Each of these factors may have an impact on the outcome and it may also have an impact on the center's decision to implement the fine system.

(9) (**a**) The equation could be written as E[Y]=20+4T+2X. (**b**) With T=0 and X=0, the expected productivity is equal to 20. (**c**) With T=1 and X=10, the expected productivity is equal to 20+4(1)+2(10)=44. (**d**) The piece rate workers are 4 units more productive than the salary workers. This relationship is statistically significant as the t-statistic is 4/1.0=4 is significantly larger than 2. (**e**) Workers with more years of experience are more productive than workers with fewer years of experience. Specifically, for each additional year of experience, the worker's productivity increases on average by 2 units. This relationship is statistically significant as the t-statistic (2/0.5=4) exceeds 2.