Problem Set 2

Insurance and Incentives

Main Points

- Twin goals of incentives and risk insurance. When the outcome of the agent's actions depends in part on factors the agent cannot control, the optimal contract must provide right incentives and optimal risk sharing.
- Benefits of Risk Sharing. In general, the risk of each party can be reduced when it is shared optimally between the principal and the agent.
- The optimal type of risk sharing depends on risk preferences. Specifically, the share of agent's pay tied to the realized outcome is lower the more risk-averse is the agent compared to the principal.

Main Concepts

Uncertainty; Fixed pay; Variable pay; Risk; Risk sharing; Expected utility; Coefficient of absolute risk aversion; Risk premium; Certainty equivalent; Risk aversion; Risk neutrality.

Problems

(1) Suppose that Sean's preferences can be described by u(y), where u is the utility function with the property that $-u''(y)/u'(y)=0.125$ and y is a random variable (e.g. income) that takes the value of 1 or 9 with equal probability. Find Sean's risk premium and certainty equivalent.

(2) Suppose that Kim's preferences can be described by $u(y)=1-e^{-0.2y}$, where y is a random variable (e.g. income) distributed normally with a mean of zero and a variance of 1. What is Kim's certainty equivalent and risk premium?

(3) The number of customers a waiter can serve per day depends stochastically on her effort according to q=e+u, where u is a random variable (e.g. weather that may affect how many customers want to go to the bar) with a mean of 0 and a variance of 1 and e is her effort that can be observed by the bar owner. The waiter's cost of effort is $c(e)=0.5e^2$ and her outside option is R=0. The coefficient of risk aversion is 0.2 for the waiter and 0.2 for the bar owner. Suppose that the waiter's pay consists of the base salary (a) and tips (bq). What contract [e, a, b] should the bar owner offer?

(4) The number of people visiting a website depends stochastically on the effort of web designer according to q=10e+u, where u is a random variable with a mean of 0 and a variance of 2 and e is her effort that can be observed by the firm. The designer's cost of effort is $c(e)=e^2$ and his outside option is R=0. The coefficient of risk aversion is 0.3 for the designer and 0.1 for the firm. Suppose that the designer's pay consists of the fixed part for designing the website (a) and a bonus (b) for each visit to the website. Suppose

that the firm decided to offer [e,b,a]=[5,0.5,0.2]. Discuss whether the firm can design a better contract.

(5) The number of books a publisher can sell depends stochastically on the writer's effort according to according to $q=10\ln(e)+u$, where u is a random variable with a mean of 0 and a variance of 1 and e is writer's effort that can be observed by the publisher. The writer's cost of effort is $c(e)=5e$, her outside option is $R=0$, and her coefficient of risk aversion is 0.2. Suppose that the writer's pay consists of an advance (a) from the publisher plus a royalty (b) for each book sold. If you know that the efficient contract involves $b^*=0.8$, what can you say about the publisher's coefficient of risk aversion?

(6) Use the agent's participation constraint in a model where the outcome is stochastically related to the agent's action and the agent's pay depends linearly on the outcome to discuss why some employees earn more than others.

Suggested Solutions

(The solutions are intended to be accurate and as complete as possible. Please report any remaining errors to jasmin.kantarevic@oma.org.)

(1) Sean's certainty equivalent can be approximated by E[y]-0.5rVar[y], where r is the coefficient of absolute risk aversion, equal to $-u''(y)/u'(y)=0.125$. The expected value of y is E[y]=0.5(1)+0.5(9)=5 and its variance is therefore $0.5(1-5)^2+0.5(9-5)^2=16$. Therefore, Sean's certainty equivalent is 5-(0.5)(0.125)(16)=4. On the other hand, Sean's risk premium can be approximated by 0.5rVar[y], which in this problem is equal to (0.5)(0.125)(16)=1.

(2) Kim's certainty equivalent can be approximated by E[y]-0.5rVar[y], where r is the coefficient of absolute risk aversion. Given that E[y]=0 and Var[y]=1, the certainty equivalent is -0.5r. Now, $r = -u''(y)/u'(y)$. Given $u(y)=1-e^{-0.2y}$, we have that $u'(y)=0.2e^{-0.2y}$ and $u''(y)=-0.04e^{-0.2y}$. Therefore, r=-(-0.04)/0.2=0.2 and CE=-0.5(0.2)=-0.1. The risk premium is 0.5rVar[y]=0.5(0.2)(1)=0.1. (Note: recall that $\partial e^{ax}/\partial x=ae^{ax}$, where a is a constant.)

(3) The problem is to maximize the owner's expected payoff subject to the waiter's participation constraint. The participation constraint for the waiter is $E[w]-c(e)-RP^A \geq R$. Now, w=a+bq, with E[w]=a+be and $Var[w]=b^2\theta$. Also, $RP^A=0.5rVar[w]=0.5rb^2\theta$. Therefore, the participation constraint becomes: $a+be-0.5e^2-0.5rb^2\theta=R$, or $a=R+0.5e^2+0.5rb^2\theta-be$. Substituting in for values of r=0.2, $\theta=1$ and R=0, we have that $a=0+0.5e^2+0.5(0.2)b^2(1)-be=0.5e^2+0.1b^2-be$. Next, the owner's expected payoff can be approximated by $E[q-w]-RP^P=E[q-a-bq]-0.5sVar[q-w]=(1-b)e-a-0.5sVar[q-w]=(1-b)e-a-0.5s(1-b)^2\theta$. Substitute for s=0.2, $\theta=1$, and for the base salary from the participation constraint to get $e-0.5e^2-0.1(1-b)^2-0.1b^2$. The first-order condition for e yields 1-e=0, from which it follows that e*=1. The first-order condition for b yields 0.2(1-b)-0.2b=0 from which it follows that b*=0.5. Lastly, use e*=1 and b*=0.5 in the participation constraint to obtain $a^*= 0.5e^2+0.1b^2-be=0.5(1^2)+0.1(0.5^2)-0.5(1)=0.025$. Therefore, the efficient contract is [e*,a*,b*]=[1,0.5,0.025].

(4) To decide whether the contract is an efficient contract, we can follow similar steps as in problem 3. Specifically, the problem is to maximize the firm's expected payoff subject to the designer's participation constraint. The participation constraint for the designer is $E[w]-c(e)-RP^A=R$. Now, w=a+bq, with E[w]=a+b10e and $Var[w]=b^2\theta$. Also, $RP^A=0.5rVar[w]=0.5rb^2\theta$. Therefore, the participation constraint becomes: $a+b10e-e^2-0.5rb^2\theta=R$, or $a=R+e^2+0.5rb^2\theta-b10e$. Substituting in for values of r=0.3, $\theta=2$ and R=0, we have that $a=0+e^2+0.5(0.3)b^2(2)-b10e=e^2+0.3b^2-b10e$. Next, the firm's expected payoff can be approximated by $E[q-w]-RP^P=E[q-a-bq]-0.5sVar[q-w]=(1-b)10e-a-0.5sVar[q-w]=(1-b)10e-a-0.5s(1-b)^2\theta$. Substitute for s=0.1, $\theta=2$, and for the base salary from the participation constraint to get $10e-e^2-0.1(1-b)^2-0.3b^2$. The first-order condition for e yields 10-2e=0, from which it follows that e*=5. The first-order condition for b yields 0.2(1-b)-0.6b=0 from which it follows that b*=0.25. Lastly, use e*=5 and b*=0.25 in the participation constraint to obtain $a= e^2+0.3b^2-b10e =(5^2)+0.3(0.25^2)-0.25(10)(5)\approx 12.5$. Therefore, the efficient contract is [e*,b*,a*]=[5,0.25,12.5]. This contract is better than the firm's offer of [5,0.5,0.2]. To see this, note that E[V] with the contract [5,0.25,12.5] is equal to $(1-b)10e-a-0.5s(1-b)^2\theta = (1-0.25)10(5)-12.5-0.5(0.1)(1-0.25)^2(2) = 24.94$, while E[V] with the contract [5,0.5,0.2] is equal to $(1-0.5)10(5)-0.2-0.5(0.1)(1-0.5)^2(2)=24.78$.

(5) To find the publisher's coefficient of risk aversion, we can proceed as in the previous two questions, treating s as the unknown. Once we find b as a function of s, we can then use $b^*=0.8$ to solve for s. The problem is to maximize the publisher's expected payoff subject to the writer's participation constraint. The participation constraint for the writer is $E[w]-c(e)-RP^A=R$. Now, $w=a+bq$, with $E[w]=a+b10\ln(e)$ and $Var[w]=b^2\theta$. Also, $RP^A=0.5rVar[w]=0.5rb^2\theta$. Therefore, the participation constraint becomes: $a+b10\ln(e)-5e-0.5rb^2\theta=R$, or $a=R+5e+0.5rb^2\theta-b10\ln(e)$. Substituting in for values of $r=0.2$, $\theta=1$ and $R=0$, we have that $a=0+5e+0.5(0.2)b^2(1)-b10\ln(e)=5e+0.1b^2-b10\ln(e)$. Next, the firm's expected payoff equals $E[q-w]-RP^P=E[q-a-bq]-0.5sVar[q-w]=(1-b)10\ln(e)-a-0.5sVar[q-w]=(1-b)10\ln(e)-a-0.5s(1-b)^2\theta$. Substitute for $\theta=1$ and for the base salary from the participation constraint to get $10\ln(e)-5e-0.5s(1-b)^2-0.1b^2$. The first-order condition for b yields $s(1-b)-0.2b=0$ from which it follows that $b^*=s/(0.2+s)$. Given that $b^*=0.8$, it follows that $s=0.8$. Therefore, the publisher is risk averse with a coefficient of absolute risk aversion equal to 0.8.

(6) The agent's participation constraint can be expressed as $E[w]=R+c(e)+0.5rb^2\theta$. Therefore, employee A is expected to earn more than employee B, if they are paid using the same payment contract (i.e. same b), under the following circumstances: A has better outside option than B ($R_A>R_B$); A works more or harder than B ($e_A>e_B$); A is more risk averse than B ($r_A>r_B$); and A's job is more risky than B's job ($\theta_A>\theta_B$).