Problem Set 10

Tournaments

Main Points

- Tournaments: Competition between agents can provide powerful incentives. In general, the agents' actions are higher the higher the gain from winning and the more important are their actions relative to luck in determining the winner.
- Advantages and Disadvantages of Tournaments: Competition between agents can induce the efficient outcome. Further, this payment method reduces measurement costs, filters out common risks, and is robust to technological changes. However, tournaments discourage cooperation and participation by disadvantaged groups, which may limit their use in practice.

Main Concepts

Tournaments; Prize spread; Relative evaluation.

Problems

(1) Curling is one of the largest amateur sports in Canada with over one million active curlers. The most popular curling tournament is the Scotties Tournament of Hearts. The final game last year was between Team Alberta and Team Manitoba. Let q be the entertainment value of watching the game, which depends on the total number of points each team scores ($q=q_{AB}+q_{MB}$). The number of points each team scores depends on the team's effort e_i according to $q_i=e_i+u_i$, where i=AB, MB and u_i is a random variable with a mean of zero. The random variable $u_{AB}-u_{MB}$ is distributed uniformly on [-2,2]. The cost of effort to each team is $0.5e_i^2$. Both teams are risk neutral. What is the optimal prize spread between winning and losing that would induce both teams to choose the efficient level of e?

(2) Chicken raised for meat are called broilers[1]. Most broilers are produced by contract growers who are paid per pound of live broiler produced. The price per pound is determined by a tournament based on each grower's 'settlement' cost, which is the sum of feed and medical costs. Consider the following tournament between two growers. The settlement cost savings are given by $q_1=e_1+0.5u$ and $q_2=e_2-0.5u$, where u is a random variable with a mean of zero that is distributed uniformly on [-1, 1]. Suppose that the cost of effort is $0.5e_i^2$ for both growers i=1,2. The grower with the higher cost savings gets P per pound of live broiler while the grower with the lower cost savings gets p, where P>p.

[1] For empirical analysis of tournaments in broiler production, see Charles Knoeber, "A Real Game of Chicken: Contracts, Tournaments, and the Production of Broilers", *Journal of Law, Economics, &Organization*, Vol. 5, No. 2. (Autumn, 1989), pp. 271-292.

What is the optimal spread P-p? Who is more likely to win the tournament, grower 1 or grower 2?

(3) A tournament is called fair and even if the contestants are identical and the rules don't favour one contestant over the other. For example, suppose that a man and a woman compete for a promotion. Assume that $q_M=e_M+0.5u$, $q_W=e_W-0.5u$, $c(e_i)=0.5e_i^2$, u is distributed uniformly on [-1,1], both contestants are risk-neutral, and both have the outside option of R=9.5. The tournament rule is that the man wins if $q_M>q_W$; otherwise, the woman wins. What is the optimal effort level for each contestant? What are the winning and losing prizes, W and w, required to make the tournament attractive for both contestants?

(4) A tournament is called even but unfair when the contestants are identical but the rules favour one over the other. Consider the model in question 4, but now assume that the tournament rule is that the man wins if $q_M>q_w-k$, where k=0.5; otherwise, the woman wins. Therefore, this tournament favours the man over the woman. What is the optimal effort level for each contestant in this case? What are the winning and losing prizes, W and w, required to make the tournament attractive for both contestants?

(5) Ronald Ehrenberg and Michael Bognanno[2] (EB) use data from the 1987 European Men's Professional Golf Association Tour to test implications of the tournament theory. Their results are presented in the following table:

Dependent variable = Final Score for the 1987 PGA European Tour

Variable	Description	Coefficient	t-value
TPRIZE	Total tournament prize money	-0.050	10.6
MAJ	Major tournament (=1 if yes)	-1.177	2.0
PAR	Par for the tournament course	2.411	6.0
YARDS	Course yardage	0.004	2.4
SAVE	Player's average score on all rounds during the 1987 tour	3.026	16.9

Note: In golf, a lower final score reflects better performance. PAR and YARDS are controls for the difficulty of the course; SAVE is a proxy for player's ability.

(a) Are these results consistent with the tournament theory?
(b) Why do Ehrenberg and Bognanno attempt to control for the player's ability (SAVE)? How would interpretation of the coefficient of TPRIZE change if they omitted SAVE?

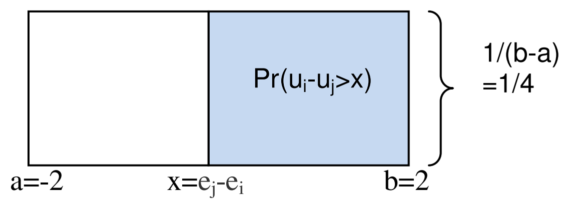
[2] Ronald Ehrenberg and Michael Bognanno, "The Incentive Effects of Tournaments Revisited: Evidence from the European PGA Tour", *Industrial and Labor Relations Review,* Vol. 43, No.3, (Feb, 1990), pp. 74S-88S.

Suggested Solutions

(The solutions are intended to be accurate and as complete as possible. Please report any remaining errors to jasmin.kantarevic@oma.org).

(1) We can approach this problem in three steps. First, the efficient level of effort for each team satisfies the first-order condition $E[q'(e^*)]=c'(e^*)$, which in this case implies $e^*=1$ for both teams since they have identical production and cost of effort functions. Second, the expected payoff for team *i* is $E[U]=w+P(e_i,e_j)*(W-w)- 0.5e_i^2$. Therefore, the first-order condition for e_i is $\partial P(e_i,e_j)/\partial e_i*(W-w)- e_i=0$. This is the incentive compatibility constraint. To find $\partial P(e_i,e_j)/\partial e_i$, note that $P(e_i,e_j)=Prob(q_i>q_j)=Prob(u_i-u_j>e_j-e_i)$. Given that u_i-u_j has the uniform distribution defined on [-2,2], we have that $P(e_i,e_j)=(1/4)(2-e_j+e_i)$ – see the diagram below - and therefore $\partial P(e_i,e_j)/\partial e_i=1/4$. Therefore, the IC becomes $(1/4)*(W-w)=e$. Third, to induce the efficient level of $e^*=1$, the IC becomes $W-w=4$.

$Prob(u_i-u_j>x)=(b-x)/(b-a)$



(2) To find the optimal spread P-p, we can proceed as in Problem (1). First, the efficient level of effort for each grower satisfies the first-order condition $E[q'(e^*)]=c'(e^*)$, which in this case implies $e^*=1$ for both growers since they have identical production and cost of effort functions. Second, the expected payoff for grower 1 is $E[U]=p+Pr(e_1,e_2)*(P-p)- 0.5e_1^2$. Therefore, the first-order condition for e_1 is $\partial Pr(e_1,e_2)/\partial e_1*(W-w)-e_1=0$. This is the incentive compatibility constraint. To find $\partial Pr(e_1,e_2)/\partial e_1$, note that $Pr(e_1,e_2)=Prob(q_1>q_2)=Prob(e_1+0.5u> e_2-0.5u)=Prob(u> e_2-e_1)$. Given that u has the uniform distribution defined on [-1,1], we have that $Pr(e_1,e_2)=(1/2)(1-e_2+e_1)$ and therefore $\partial Pr(e_1,e_2)/\partial e_1=1/2$. Therefore, the IC becomes $(1/2)*(P-p)=e_1$. Third, to induce the efficient level of $e^*=1$, the IC becomes $P-p=2$. Since the two growers are identical in terms of production and cost of effort, they will both supply the same efficient level of effort, $e^*=1$. Therefore, the probability that grower 1 wins is $Pr(e_1,e_2)=(1/2)(1-e_2+e_1)=(1/2)(1-1+1)=1/2$. Similarly, the probability that grower 2 wins is $1-Pr(e_1,e_2)=1-1/2=1/2$. Therefore, each grower is equally likely to win.

(3) Since the man and woman are identical in terms of the production and cost of effort functions, we can consider the man's problem, as the identical analysis applies for the woman. First, the efficient level of effort is 1. Second, the expected payoff is $E[U]=w+Pr(e_M,e_w)(W-w)- 0.5e_M^2$. The first-order condition is $\partial P(e_M,e_W)/\partial e_M*(W-w)- e_M=0$. This is the incentive compatibility constraint. To find $\partial P(e_M,e_W)/\partial e_M$, note that $P(e_M,e_W)=Prob(q_M>q_W)=Prob(u>e_W-e_M)$. Given that u has the uniform distribution defined on [-1,1], we have that $P(e_M,e_W)=(1/2)(1-e_W+e_M)$ and therefore $\partial P(e_M,e_W)/\partial e_M=1/2$. Therefore, the IC becomes $(1/2)*(W-w)=e$. Third, to induce the efficient level of $e^*=1$, the IC becomes $W-w=2$. In addition, W and w must satisfy the participation constraint $E[U]= w+Pr(e_M,e_w)(W-w)- 0.5e_M^2=w+0.5(2)-0.5=w+0.5=R=9.5$, since $Pr(e_M,e_w)=0.5$ as both the man and woman choose the same effort level and have identical probability for winning. This implies that $w=9$. From IC, we have that $W=w+2$, so $W=11$.

(4) As in problem (3), the efficient level of effort is 1 for both man and woman. Moreover, the expected payoff for the man is $E[U]_M = w + Pr(e_M, e_w)(W - w) - 0.5e_M^2$ and for the woman it is $E[U]_W = w + [1 - Pr(e_M, e_w)](W - w) - 0.5e_W^2$. The first-order conditions are therefore $\partial P(e_M, e_W)/\partial e_M * (W - w) - e_M = 0$ for the man and $-\partial P(e_M, e_W)/\partial e_W * (W - w) - e_W = 0$ for the woman. In addition, $Pr(e_M, e_w) = Pr(q_M > q_w - 0.5) = Pr(e_M + 0.5u > e_W + 0.5u - 0.5) = Pr(u > e_W - e_M - 0.5) = (1/2)(1 - e_W + e_M + 0.5)$. Therefore, $\partial P(e_M, e_W)/\partial e_M = 1/2$ and $-\partial P(e_M, e_W)/\partial e_W = 1/2$. Therefore, the IC is identical for both man and woman, $(1/2)(W - w) = e^* = 1$, which implies that the optimal prize spread is $W - w = 2$. Now, with $e_W = e_M = 1$, we have that $Pr(e_M, e_w) = (1/2)(1 - e_W + e_M + 0.5) = 0.75$ and $1 - Pr(e_M, e_w) = 0.25$. Therefore, the expected payoff for the woman is $w + 0.25(2) - 0.5 = w = R = 9.5$. From the IC, we have that $W = w + 2$, so $W = 9.5 + 2 = 11.5$. We also have to verify that $w = 9.5$ and $W = 11.5$ satisfy the man's participation constraint. The man's expected payoff with these prizes is $9.5 + 0.75(2) - 0.5 = 9.5 + 1 = 10.5 > R = 9.5$, so the participation constraint is satisfied for the man as well.

(5) (**a**) The tournament theory predicts that participants in the tournament will 'work' harder when the prize spread is higher. The results suggest that this may be the case: the final score is significantly smaller (t-value >2) both when the winning prize is higher and when it is a major tournament. However, the tournament theory claims that participants will work harder because of better incentives. From the results presented in the table, it is difficult to say whether they represent incentive or selection effects. (**b**) EB control for SAVE because it can influence both the outcome (the final score) and who participates in the tournament. Without controlling for SAVE, it would be more difficult to say whether the final score in tournaments with higher prizes is lower because players try harder or because more able players participate in the tournament.