Problem Set 1

Basic Incentive Contract

Main Points

- Employment relationships can often be conveniently described using the Principal-Agent Framework. The main elements include: the parties, the agent's action, the outcome, the cost of action, the payoff of each party, and the outside option for each party.
- (Pareto) Efficient Contract: An efficient contract is defined as a contract that maximizes the payoff of at least one party without making the other party worse off.
- Efficiency and MB=MC Principle: To maximize the gain from the relationship, both benefits and costs must be considered. Technically, the actions that maximize the gain from the relationship equate their incremental costs and benefits.
- Not all relationships should be formed: To determine whether a relationship should be formed, the maximum gain from the relationship should be compared to outside options.
- Equity: As long as it is efficient to form the relationship, there exists a compensation package that is acceptable to both parties. The exact level of compensation will in general depend on each party's bargaining power.

Main Concepts

Principal-agent relationship; Agent's action; Outcome; Reward/Payment; Agent's cost of action; Payoff; Outside option; Contract; Social surplus; Efficiency; Equity; Bargaining power; 'Take-it-or-leave-it' offer; First mover advantage; Participation constraint.

Problems

(1) Ana needs to hire a contractor to renovate her house. For each hour of contractor's work e, the value of Ana's house increases by $10e$. The contractor's disutility of work is $0.5e^2$. Ana's outside option is 0 and the contractor's outside option is 25. Assume that Ana's payoff is the difference between the value of her house and the payment to the contractor, and that the contractor's payoff is the payment net of his disutility of work. Is it efficient that Ana hires the contractor? If so, what contract should Ana offer to the contractor?

(2) Bill wishes to hire a stockbroker to manage his portfolio. For each unit of stockbroker's effort e, the value of Bill's portfolio increases by $10\ln(e)$. The stockbroker's disutility of effort is $5e$. Bill's outside option is 0 and the stockbroker's outside option is 20. Assume that Bill's payoff is the value of his portfolio minus the payment to the stockbroker, and that the stockbroker's payoff is the payment net of his disutility of effort. Is it efficient that Bill hires the stockbroker? If so, what contract should the stockbroker offer to Bill?

(3) Claudette wants to hire a financial planner to prepare her retirement plan. For each unit of planner's effort e, the value of Claudette's retirement plan increases by $5e$. The planner's disutility of effort is $5e^2$. Claudette's outside option is 0.25 and the planner's outside option is 0.5. Assume that Claudette's payoff is the value of her retirement plan net of the payment to

the planner, and that the planner's payoff is his payment net of disutility of effort. Which of the following two contracts, if any, is efficient: $[e_1,w_1]=[0.5,\$2]$ or $[e_2,w_2]=[1,\$2.25]$?

(4) A physician can treat 20 patients for each hour of work *e*. The value of each treated patient to the Ministry of Health is 30. The physician's disutility of work is given by 500*e*. What number of hours should the physician work if the medical regulations stipulate that the physician can work no more than 8 hours per day? Assume that the outside options are such that it is efficient for the Ministry of Health to employ the physician.

(5) A nurse can treat one patient for each hour of work *e*. The value of each treated patient to the hospital is 1. The nurse's disutility of work is $0.5e^2$ if she works one hour or less and e^2 if she works more than one hour. What is the efficient level of *e*? Assume that the outside options are such that it is efficient for the hospital to employ the nurse.

(6) A worker produces output according to $q(e)=10e$, where *e* is effort. The worker's cost of effort is $0.5e^2$. If *e* can take only four values: {0, 1, 2, 3), what is the efficient level of *e*? Assume that the outside options are such that it is efficient that the firm employs the worker.

Suggested Solutions

(These solutions are intended to be accurate and as complete as possible. Please report any remaining errors to jasmin.kantarevic@oma.org.)

(1) In this case, Ana is the principal and the contractor is the agent. To decide whether Ana should hire the contractor, we can proceed in two steps. First, we find the contract that maximizes principal's payoff and is also acceptable to the agent. Second, we verify that the principal is no worse by hiring the agent than by not hiring him. Step 1: The efficient contract [e, w] maximizes Ana's payoff V=q-w=10e-w. To be acceptable to the contractor, the contract must satisfy the contractor's participation constraint U≥R, where U=w-c(e)=w-0.5e^2 and R=25. Therefore, w that satisfies the contractor's participation constraint is w≥R+c(e)=25+0.5e^2. Given that Ana designs the contract, this constraint will be binding because of the first-mover advantage (U=R). Substituting for w in Ana's payoff yields V=10e-w=10e-25-0.5e^2. This is now Ana's payoff that satisfies the contractor's participation constraint. Notice that this payoff depends only on the choice of e. The first-order condition for e is 10-e*=0, which implies that e*=10. Substituting for e*=10 in the contractor's participation constraint gives w*=25+0.5(10^2)=75. Therefore, the contract that maximizes Ana's payoff and is also acceptable to the contractor is [e*,w*]=[10,75]. Step 2: We'll verify that V(e*,w*) is at least as great as Ana's outside option S=0. We have that V(e*,w*)=10e*-w*=10(10)-75=25. This is greater than Ana's outside option of 0. Therefore, it is efficient that Ana hires the contractor.

(2) In this case, Bill is the principal and the stockbroker is the agent. We can proceed as in the previous problem, except that now we wish to find a contract that maximizes the agent's payoff but is still acceptable to the principal. Step 1: The efficient contract [e, w] maximizes the stockbroker's payoff U=w-c(e)=w-5e. To be acceptable to Bill, the contract must satisfy Bill's participation constraint V≥S, where V=q-w=10ln(e)-w and S=0. Given that the agent designs the contract, the participation constraint for the principal will be binding (i.e. V=S). Therefore, w that satisfies Bill's participation constraint is w=q(e)-S=10ln(e). Substituting for w in the stockbroker's payoff yields U=w-c(e)= 10ln(e)-5e. This is now the stockbroker's payoff that satisfies Bill's participation constraint. Notice that this payoff depends only on the choice of e. The first-order condition for e is 10/e*-5=0, which implies that e*=2. Substituting for e*=2 in Bill's participation constraint gives w*=10ln(2)≅6.9. Therefore, the contract that maximizes the stockbroker's payoff but is still acceptable to Bill is [e*,w*]=[2,6.9]. Step 2: We'll verify that U(e*,w*) is at least as great as R=20. We have that U(e*,w*)=w*-5e*=6.9-5(2)=-3.1. Since this is smaller than the stockbroker's outside option of 20, it is not efficient that Bill hires the stockbroker.

(3) In this case, Claudette is the principal and the financial planner is the agent. The problem does not specify who designs the contract, but we know that the choice of the efficient level of effort does not depend on who designs the contract. To find the efficient level of effort, we have to find e that maximizes the value of the relationship, q(e)-c(e)=5e-5e^2. The first-order condition is 5-10e*=0 and therefore e*=0.5. To determine the payment acceptable to both Claudette and the planner, we use the participation constraint for each party. For Claudette, the constraint is that V*≥S, or 5e*-w*≥0.25, or w*≤5(0.5)-0.25=2.25. For the financial planner, the constraint is that U*≥R, or w*-c(e*)=w*-5e*^{2}≥0.5, or w*≥0.5+5(0.5^2)=1.75. Therefore, the efficient contract has e*=0.5 and 1.75≤w*≤2.25. Therefore, the contract [e_1,w_1]=[0.5,$2] is efficient, while the contract [e_2,w_2]=[1,$2.25] is not efficient.

(4) In this case, the physician is the agent and the Ministry of Health is the principal. To find the efficient number of hours of work, we have to compare the marginal benefit and the marginal cost

of hours of work. The total benefit is q(e)= 30×20e=600e, so the marginal benefit is 600. The total cost is 500e, so the marginal cost is 500. Therefore, the marginal benefit always exceeds the marginal cost, for any value of e. Therefore, the physician should work the maximum number of hours permitted by the medical regulations, 8. Note that in this case we cannot use the first-order condition $q'(e^*)-c'(e^*)=0$ to find the efficient level of effort because both the total benefit and the total cost functions are linear ($q''=0=c''$ for all e).

(5) In this case, the hospital is the principal and the nurse is the agent. To find the efficient number of hours of work, we have to find e that maximizes the value of the relationship, q(e)-c(e). This value is $e-0.5e^2$ for $e\leq1$ and $e-e^2$ for $e>1$. In other words, this value is defined over two intervals of e. To find which e maximizes this value, we have to find in the first step the optimal e for each interval. For $e\leq1$, the first-order condition gives $1-e^*=0$, or $e^*=1$. Therefore, the value of the relationship is $e-0.5e^2=1-0.5(1^2)=0.5$. For $e>1$, the value of the relationship is $e-e^2=e(1-e)$, which is negative for any $e > 1$. Therefore, the efficient level of effort is $e^*=1$.

(6) The value of relationship is $q(e)-c(e)=10e-0.5e^2$. To find efficient e, we have to evaluate the relationship value at each distinct value of e. We have: $q(0)-c(0)=0$; $q(1)-c(1)=10-0.5(1^2)=9.5$, $q(2)-c(2)=20-0.5(2^2)=18$, and $q(3)-c(3)=30-0.5(3^2)=25.5$. Therefore, the efficient value of e is 3. Note that this choice is restricted because e can take only these four discrete values. If e was a continuous variable, the first-order condition for e is $10-e=0$, and the efficient level of e would therefore be 10.