UNIVERSITY OF TORONTO
DEPARTMENT OF ECONOMICS

ECONOMICS 381H5S – SUMMER 2009

MANAGERIAL ECONOMICS II: PERSONNEL ECONOMICS

Midterm 2
SOLUTIONS

Instructions

The test is 50 minutes long. Non-programmable calculators are allowed. The test consists of four questions, each worth 5 points. Show all your work in the space provided below the question. If you need additional space, you may write on the back of the page.

LAST NAME _____

FIRST NAME _____

STUDENT NUMBER _____

GOOD LUCK!

Question 1	Question 2	Question 3	Question 4	Total
/5	/5	/5	/5	/20

1. The average math score of students in Trudeau High School in Ottawa is given by $q=e+u$, where e is teacher's effort and u is a random variable with a mean of 0 and a variance of 2. The teacher's disutility of effort is $c(e)=e^2/8$. The teacher is risk averse, with the coefficient of absolute risk aversion equal to 3. The school principal is risk-neutral. What is teacher's risk premium if the school principal designs an optimal piece rate contract?

- Teacher's risk premium is $0.5rVb^2=0.5(3)(2)b^2=3b^2$. To evaluate this, we need to find optimal b.
- The teacher's certainty equivalent is $a+be-0.5rVb^2-e^2/8$. The incentive compatibility constraint is therefore $b-e/4=0$ or $e=4b$.
- The principal maximizes the expected profit subject to ICC and PC. The expected profit is $e- 0.5rVb^2-e^2/8= 4b-0.5rVb^2-(4b)^2/8$. The first-order condition for b is $4-rVb-4b=0$, from which it follows that $b=4/(4+rV)=4/(4+3\times2)=0.4$.
- Therefore, teacher's risk premium if the school principal designs an optimal piece rate contract is $3(0.4)^2=0.48$.

2. Employee's contribution to the firm is given by $q=e+u$, where e is employee's effort and u is a random variable with a mean of zero. The cost of effort function is $e^2/6$. Both the employee and employer are risk neutral and have an outside option of zero. Compare the value of the expected social surplus when the employer uses q as a performance measure and when the employer uses $y=kq$ as a performance measure, where k is a parameter that the employee can manipulate. The employee knows that the actual value of k is 1, while the employer knows only that k has a mean of 1 and a variance of $V=1$.

- When the employer uses q as a performance measure, he can induce the efficient level of effort by using a piece rate contract $w=a+q$. This level of effort is given by the $MB(e)=MC(e)$ condition, which in this example is $1=e^*/3$, so $e^*=3$. The value of social surplus is then $E[q(e^*)]-c(e^*)=3-3^2/6=1.5$.
- When the employer uses y as a performance measure, the employee maximizes $a+bE[y]-c(e)=a+bke-e^2/6$. The ICC condition is then $bk-e/3=0$ or $e=3bk$.
- The employer maximizes his expected profit subject to ICC and PC. The expected profit function is $E[e]-c(E[e]) = 3bE[k]-E[(3bk)^2]/6 = 3b-1.5b^2E[k^2] = 3b-1.5b^2(1+V) = 3b-3b^2$. The optimal value of b is then 0.5. Given $b=0.5$, the employee will choose $e=3bk=1.5$.
- The social surplus when the employer uses y is then $1.5-1.5^2/6=1.125$.

3. Tropicana Products, a company owned by Pepsi that specializes in the production of orange juice, employs two orange pickers. The number of oranges picked is $q_1=e_1+0.5u$ for the first picker and $q_1=e_1-0.5u$ for the second picker, where e represents pickers'

effort and u is a random variable that is distributed uniformly on [-1, 1]. The pickers are risk-neutral and have the same cost of effort function given by $e^2/4$ and the same outside option. Tropicana currently pays a wage $w_i=q_i$ to each picker i=1,2, but this payment method has significant measurement costs because each day the company has to count the number of oranges picked by each worker. Design an alternative payment method that may lower Tropicana's measurement costs but still provide optimal incentives to pickers.

- $w_i=q_i$ is an optimal piece rate that will induce the efficient level of effort e*. The MB(e)=MC(e) conditions in this example gives e*=2.
- The value of base payment a in this example is 0. From the participation constraint, we have that $a=R+c(e)-be=R+2^2/4-1(2)=R-1$, so R=1.
- A tournament contract can induce the same efficient level of effort e*=2. The ICC condition for each picker is that e/2=f(0)(W-w) since the pickers are identical and will choose the same level of effort. Given e*=2 and f(0)=1/2 for the uniform variable u on [-1,1], the ICC becomes W-w=2.
- To find the prizes, we can use the participation constraint for any picker: w+p(W-w)-c(e)=R. p=0.5 since each picker is likely to win, so PC becomes $w+0.5(2)-2^2/4=1$. This then implies that w=1 and W=3.

4. Explain why fixed bonus pools are rarely used by companies.

- The answer is based on Bol (2005).
- Fixed bonus pools are rarely used by firms for at least three reasons:
    - They discourage co-operation among employees.
    - They encourage sabotage activities of employees.
    - They are not effective in solving other problems related to rating bias, such as favoritism and rent-seeking.