

UNIVERSITY OF TORONTO  
DEPARTMENT OF ECONOMICS

ECONOMICS 381H5S – SUMMER 2010  
**MANAGERIAL ECONOMICS II: PERSONNEL ECONOMICS**

## Midterm 1 - Solutions

### Instructions

The test is 50 minutes long. Non-programmable calculators are allowed. The test consists of four questions, each worth 5 points. Show all your work in the space provided below the question. If you need additional space, you may write on the back of the page.

LAST NAME \_\_\_\_\_

FIRST NAME \_\_\_\_\_

STUDENT NUMBER \_\_\_\_\_

GOOD LUCK!

Question 1	Question 2	Question 3	Question 4	Total
/5	/5	/5	/5	/20

1. The Hospital for Sick Children (HSC) considers hiring a new pediatrician. The pediatrician can treat  $n \times e$  patients per day, where  $n$  represents the number of nurses working with the pediatrician and  $e$  represents the pediatrician's effort. The HSC values the treatment of each patient at \$1. The pediatrician's cost of effort is  $0.5e^2$ . In addition, the outside options are 0 for the HSC and  $R$  for the pediatrician. Show that it is efficient that the HSC hires the pediatrician if and only if  $R \leq 0.5n^2$ . Assume that the HSC can observe the pediatrician's effort.

- It is efficient that the HSC hires the pediatrician if and only if  $p(q(e^*)) - c(e^*) \geq R + \Pi_R$ . (1 point),
- ... where the efficient level of effort satisfies  $p(q'(e^*)) - c'(e^*) = 0$  (1 point).
- Therefore,  $1 \times n - e^* = 0$ , so  $e^* = n$ . (1.5 points)
- Therefore, it is efficient that the HSC hires the pediatrician if and only if  $1 \times n^2 - 0.5n^2 \geq R + 0$ , which simplifies to  $R \leq 0.5n^2$ . (1.5 points)

2. The school principal considers hiring a new math teacher. The principal values each point improvement in the standardized math test score at \$2. In turn, the math score is given by  $e + u$ , where  $e$  is teacher's effort and  $u$  is a random variable with a mean of 5. The teacher's cost of effort is given by  $0.1e^2$ , while his outside option is 10. The principal's outside option is 0. Is it efficient that the principal hires the teacher? If so, what piece rate contract should the principal offer if the teacher's effort cannot be verified? Assume that both the principal and the teacher are risk neutral.

- It is efficient that the principal hires the teacher if and only if  $pE[q] - c(e) \geq R + \Pi_R$ , or  $2 \times (e^* + 5) - 0.1e^{*2} \geq 10$ . (1 point)
- The optimal level of effort  $e^*$  satisfies  $pE[q'(e^*)] = c'(e^*)$ , or  $2 \times 1 = 0.2e^*$ , which implies that  $e^* = 10$ . (0.5 point)
- Therefore,  $2 \times (10 + 5) - 0.1(10^2) = 20 > 10$ , so it is efficient that the principal hires the teacher. (0.5 point)
- The optimal piece rate  $b^*$  induces the teacher to supply the efficient level of effort  $e^*$ . The teacher's expected utility, given a contract  $[a, b]$ , is  $a + bpE[q] - c(e)$ , or  $a + b2(e + 5) - 0.1e^2$ . The first-order condition is  $2b - 0.2e = 0$ . Substituting for the optimal  $e^* = 10$ , this yields  $2b - 2 = 0$ , or  $b^* = 1$ . (1.5 point)
- The optimal fixed payment  $a^*$  satisfies the participation constraint:  $a + bpE[q] - c(e) = R$ . This yields  $a^* + b^*2(e^* + 5) - 0.1e^{*2} = a^* + 30 - 10 = a^* + 20 = R = 10$ , or  $a^* = -10$ . (1.5 point)

3. A common complaint of university students is that professors seem too remote and uninterested in teaching them. As a consequence, some universities pay their professors based on students' evaluations. A recent study has found that learning outcomes at these universities is about 40 percent higher than at universities that don't use this payment structure. Discuss whether this result is consistent with economic theory and with empirical evidence in Shearer (2002). If not, what could explain this difference?

- The results indicate that piece rate workers are more productive than salary workers. This result is consistent with the economic theory in the case where the professors' actions cannot be observed or verified. (1 point)
- The estimate of 40% is significantly larger than 20 percent productivity improvement that Shearer (2002) found for tree planters in B.C. using a randomized experiment. (1 point).
- This difference can probably be explained by the selection effect. Specifically, it is possible that professors producing lower learning outcomes moved from universities that use the students' evaluations to those universities that don't use the evaluations. Similarly, professors producing higher learning outcomes may have moved from universities that don't use the evaluations to those that do. In both cases, the impact on learning outcomes will be overstated if this selection effect is not controlled for. (2 points)
- An additional reason for the difference is that the results presented in Shearer (2002) may be specific to the tree-planting industry and these results may not extrapolate to the case of academic teaching. (1 point)

4. The worker's expected output is  $E[q]=e+n$ , where  $e$  is worker's effort and  $n$  is worker's ability. Each worker is either of low ability ( $n=0$ ) or high ability ( $n=1$ ). The worker's cost of effort is given by  $0.5e^2$ , while his outside option is 0. The value of each unit of output is \$1. The worker can work for a salary firm that pays  $s=0$  or a piece rate firm that pays  $w=-1+q$ . If workers are free to choose for which firm to work, what is the expected observed difference in productivity between salary and piece rate workers? How much of this difference is the incentive effect and how much is the selection effect? Assume that the worker's effort cannot be observed or verified.

- The worker in the salary firm would choose  $e=0$  since his pay does not depend on his effort. The worker in the piece rate firm would choose the efficient level of effort, given by the first-order condition  $pE[q'(e^*)]=c'(e^*)$ , or  $1 \times 1 = e^*$ , or  $e^*=1$ .
- The worker will choose which firm to work for by comparing his expected utility in each firm. In the salary firm, the utility is  $s-c(e)=0$ . In the piece rate firm, the expected utility is  $-1+E[e^*+n]-c(e^*)=-1+1+n-0.5(1^2)=n-0.5$ . Therefore, the expected utility in the piece rate firm is  $1-0.5=0.5$  for the high ability worker and  $0-0.5=-0.5$  for the low ability worker. Therefore, the high ability worker will choose to work for the piece rate firm, while the low ability worker will choose to work for the salary firm. (2 points)
- The expected productivity,  $E[q]=e+n$ , is then 0 for the salary workers and  $1+1=2$  for the piece rate workers. Therefore, the observed difference is 2. (1 point)
- The incentive effect is the difference in productivity for the high ability workers between their productivity as the piece rate workers,  $e+n=1+1=2$ , and their productivity as the salary workers,  $e+n=0+1=1$ . (1 point)
- Therefore, the selection effect is the difference between the observed productivity and the incentive effect,  $2-1=1$ . (1 point)