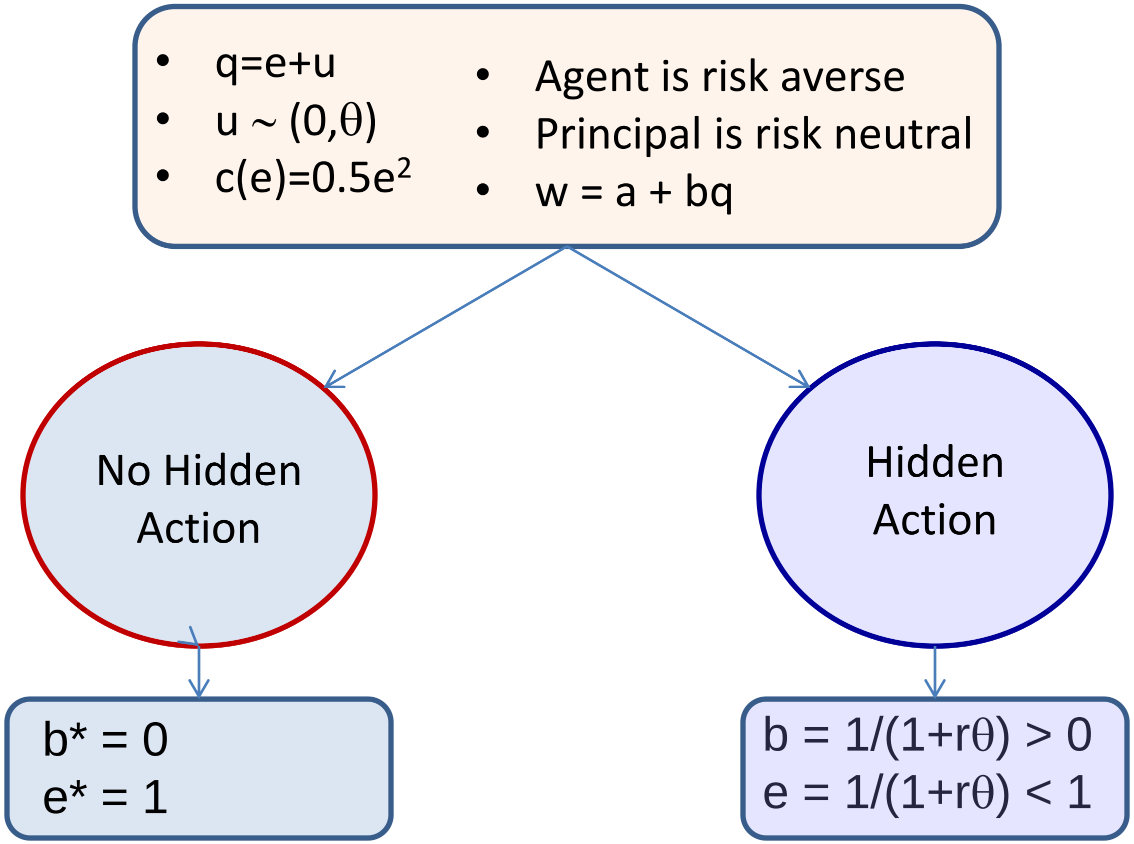
Multiple Signals

Class 5

How to Pay CEOs?

Consequences of Hidden Action



- Optimal Risk Insurance
- Optimal Incentives

- Inefficiently high b
- Inefficiently low e

Addressing Efficiency Loss

- With hidden action, there is an efficiency loss because

$$b = 1/(1+r\theta) = e < e^* = 1$$

- At least two ways to improve incentives (i.e. increase b):

1. Contract with less risk-averse agent (lower r)
2. Reduce uncertainty (lower θ)

Objectives for Today

1. Optimal Contract with Multiple Signals
2. Interpretation and Applications
3. Application: Relative Evaluation for CEOs

Additional Signal of Performance

$$w = a + bq + cy$$

Agent	q (Outcome)	y (Additional Signal)
Surgeon	Number of surgeries	Average number of surgeries by other surgeons
Fund manager	Portfolio performance	Market performance
Student	Final exam	Prior GPA
Real estate agent	Number of sold houses	Her hair colour

Two Main Questions

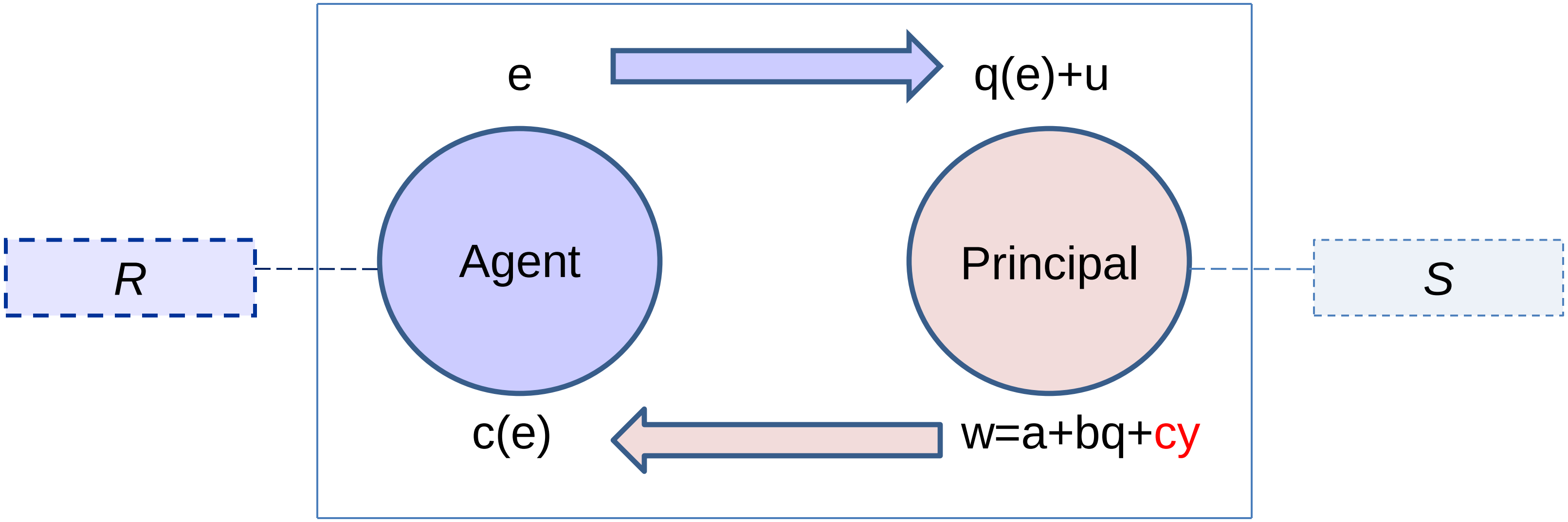
$w = a + bq + cy$

When should we use signal y?

- $c=0$ or $c \neq 0$?

How should we use signal y?

- $c>0$ or $c<0$?



Additional Assumptions

- $u \sim (0,\theta)$
- $q(e)=e$
- $c(e)=0.5e^2$
- A is risk averse
- P is risk neutral
- $R = 0 = S$
- ______________ ________________

Compensation

- $w=a+bq+cy$

$$E[w] = E[a+bq+cy]$$
$$= E[a]+E[bq]+E[cy]$$
$$= \underline{\hspace{6cm}}$$

$$Var[w] = Var[a+bq+cy]$$
$$= Var[bq]+Var[cy]+2cov(bq,cy)$$
$$= \underline{\hspace{9cm}}$$

Payoffs

- $E[w] = a + be$
- $Var[w] = b^2\theta + c^2 + 2bc\rho$

- V =E[q-w]-0.5sVar[q-w]

  = ____________________________________

- U =E[w]-0.5rVar[w]-c(e)

  = ____________________________________

The Problem

- Max E[V]
  s.t. (PC) $E[U] \geq R$
  (IC) e maximizes E[U]

- $E[V] = (1-b)e - a$

- $E[U] = a + be - 0.5r(b^2\theta + c^2 + 2bc\rho) - 0.5e^2$

Solution by Backward Induction

1. Incentive compatibility constraint

- $\text{Max}_e\ E[U] = a+be-0.5r(b^2\theta+c^2+2bc\rho)-0.5e^2$
- (IC) ______________________________

2. Participation constraint

- $E[U] = a+be-0.5r(b^2\theta+c^2+2bc\rho)-0.5e^2 = R = 0$

$\Rightarrow$ (PC) ______________________________

3. Principal's objective, subject to IC and PC

- $E[V] = (1-b)e - a$

  $= (1-b)e - (0.5r\,(b^2\theta + c^2 + 2bc\rho) + 0.5e^2 - be)$ from PC

  $= (e - 0.5e^2) - 0.5r(b^2\theta + c^2 + 2bc\rho)$

  $= (b - 0.5b^2) - 0.5r\,(b^2\theta + c^2 + 2bc\rho)$ from IC

Optimal Contract

$$\text{Max } E[V]_{b,c} = (b-0.5b^2) - 0.5r(b^2V+c^2+2bc\rho)$$

- First-order conditions:
  - $-0.5r(2c+2b\rho)=0$ for c
  - $1-b-0.5r(2b\theta+2c\rho)=0$ for b

- This implies:
  - $2c+2b\rho=0$ $\Rightarrow$ c = ____________________
  - $1-b-r(b\theta+c\rho)=0$ $\Rightarrow$ b = ____________________

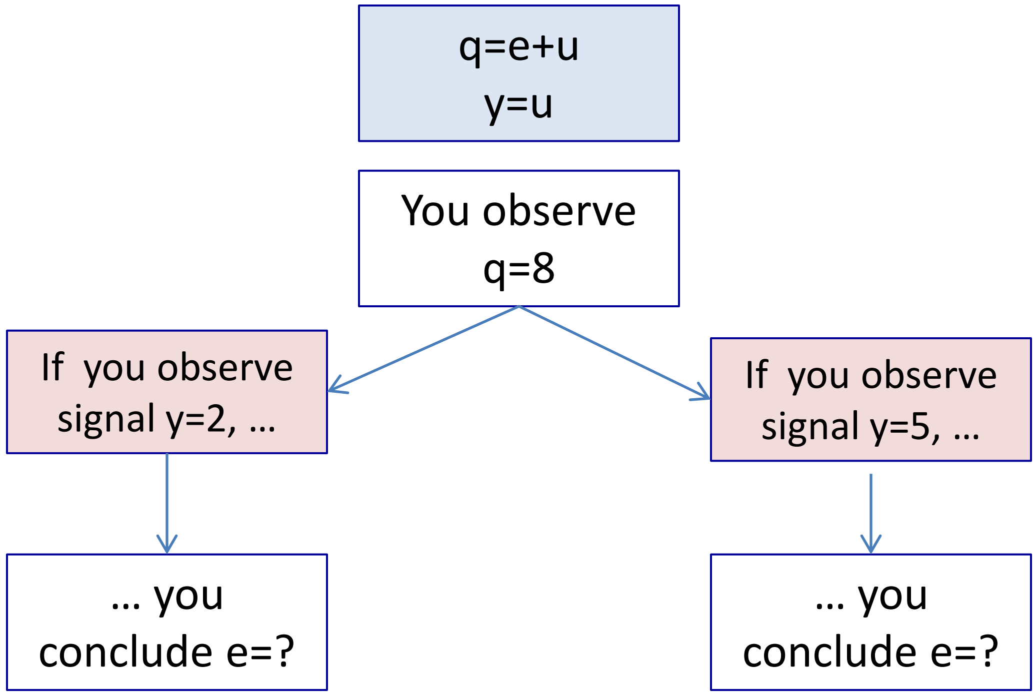
Interpretation: Weight on Signal y

$$c=-b\rho$$

1. Use signal y only if $\rho \neq 0$
   - i.e. signal is informative about performance q

2. Sign of c opposite of ρ
   - If $\rho<0$, $c>0$
   - If $\rho>0$, $c<0$

Intuition with Positively Correlated Signal (ρ>0)



- Compensation when signal is 2 is <u>higher/lower</u> then when signal is 5;
- i.e. compensation is <u>negatively/positively</u> related to signal (c<0).

Application: Adjusting Midterm Grades

- Let y be the class average
- Let q be your midterm grade
- y and q are positively correlated (**$\rho>0$**)

- Your adjusted midterm grade:
  - If y is low   ($y < 40$), your grade is q+5
  - If y is high ($y \geq 40$), your grade is q

} **$c < 0$**

Impact on Risk Premium

- RP = 0.5rVar[w]

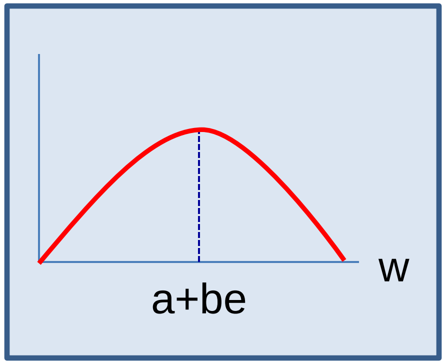
- With no additional signal (w=a+bq)
  - Var[w] $= b^2\theta$

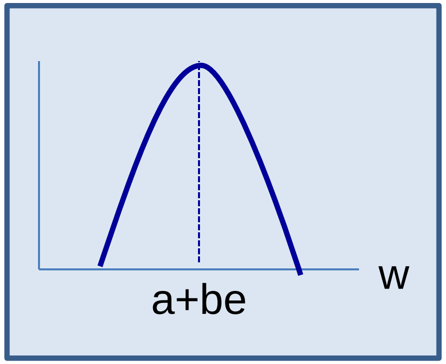
- With additional signal (w=a+bq+cy)
  - Var[w] $= b^2\theta + c^2 + 2bc\rho$

    $= b^2\theta + b^2\rho^2 - 2b^2\rho^2$

    $= b^2(\theta - \rho^2) < b^2\theta$

→ Using signals may reduce risk!

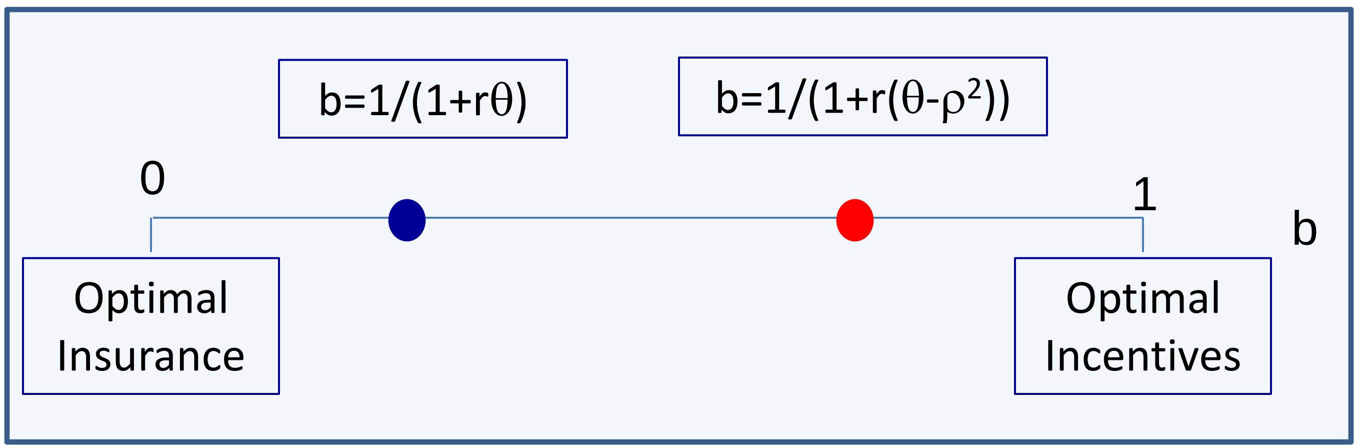




Impact on Incentives

- With no additional signal : $b=1/(1+r\theta)$
- With additional signal : $b=1/[1+r\,(\theta-\rho^2)]$

→ **Using signals may improve incentives!**



Gibbons and Murphy (1990)

- About 2,000 CEOs from about 1,300 firms, 1974 to 1986
- Empirical Model

$$w_i = \alpha + \beta q_i + \gamma y_i + \varepsilon_i$$

where:

w	Pay: change in ln(CEO salary and bonus)
q	Firm's rate of return: continuously accrued rate of return received by shareholders, including price appreciation and dividends
y	Market return: continuously accrued rate of return on the value-weighted portfolio of firms in the market

Table 1. Coefficients of Ordinary Least Squares Regressions of Δln (CEO Salary + Bonus) on Firm, Industry, and Market Rates of Return on Common Stock.
(t-Statistics in Parentheses)

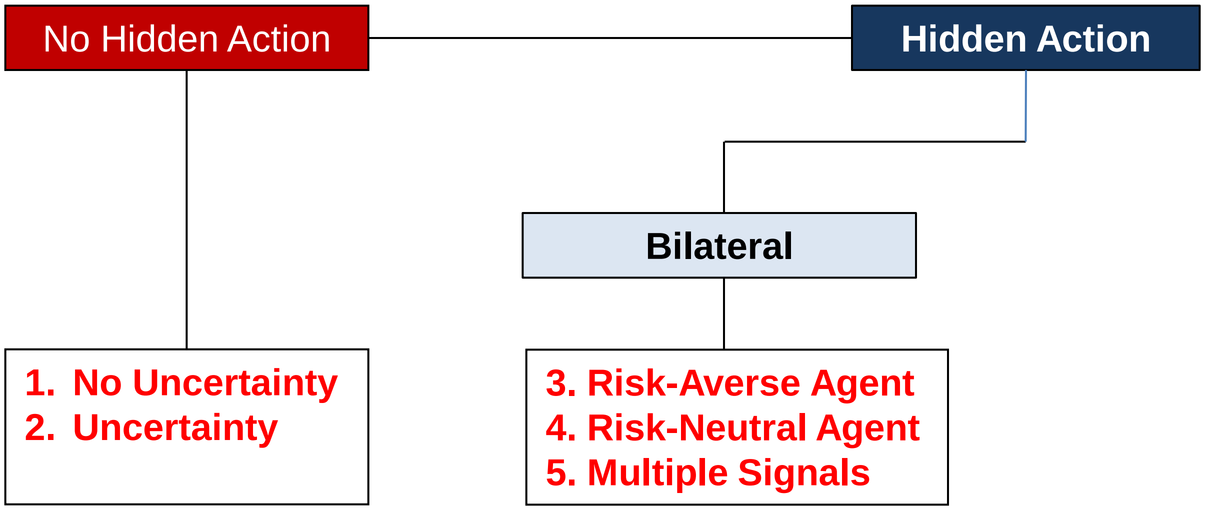
Independent Variable		*Dependent Variable: Δln(CEO Salary + Bonus)* (1)	(2)	(3)	(4)
Intercept	α	.046		.068	
Firm's Rate of Return on Common Stock	β	.1562 (19.5)		.1805 (21.0)	
Market Value-Weighted Return on Common Stock	γ			−.1490 (−7.6)	

Note: Sample size is 7,757 for all observations. The sample is constructed from longitudinal data reported in *Forbes* on 1,668 CEOs serving in 1,049 firms from 1974 to 1986.

Main Points

1. **In formativeness Principle**: An additional signal may reduce risk and improve incentives in the principal-agent model with hidden action if the signal is informative about the agent’s performance. In this case, the compensation should vary positively with the signal if the signal and performance are negatively related, and negatively if the signal and performance are positively related.

What's on the Midterm?



<u>Additional Office Hours</u>

Monday, June 2, 5-7pm, BF 323 (Bancroft Building)