

Risk Averse Agent

Class 3



Economics of Driving Taxis

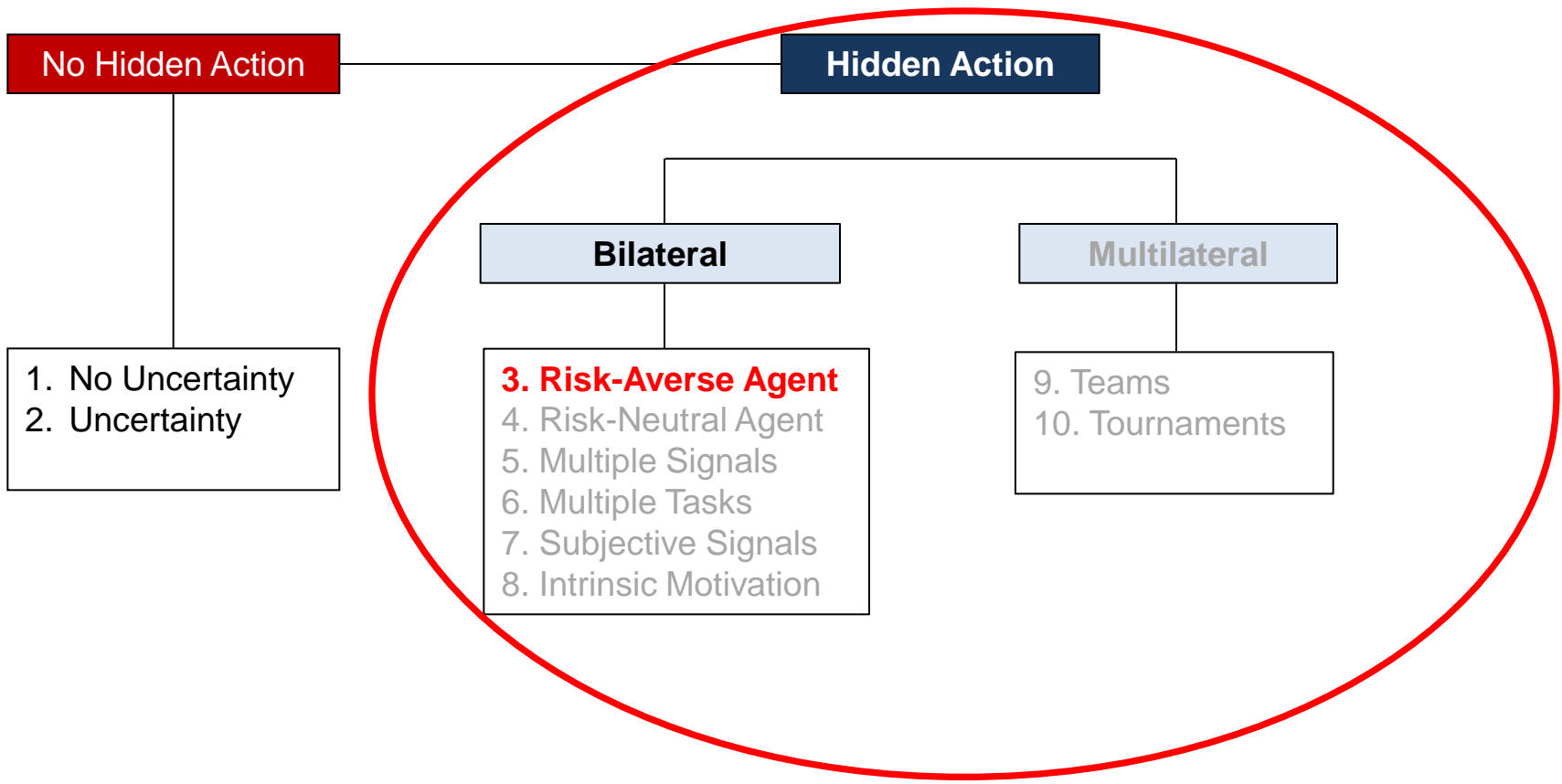
	Lease Drivers	Shift Drivers
Own Plate?	No	No
Owns Car?	Yes	No
Expenses		
<i>Dispatch</i>	Yes	Yes
<i>Car Payments</i>	Yes	
<i>Insurance</i>	Yes	
<i>Maintenance</i>	Yes	
<i>Gas</i>	Yes	Yes
<i>Lease Fee</i>	Yes	
<i>Rental Fee</i>		Yes
Income		
<i>Fares and Tips</i>	Yes	Yes
<i>Rental Fee</i>	Yes	

Shift driver pays rent to lease

Shift driver keeps all fares

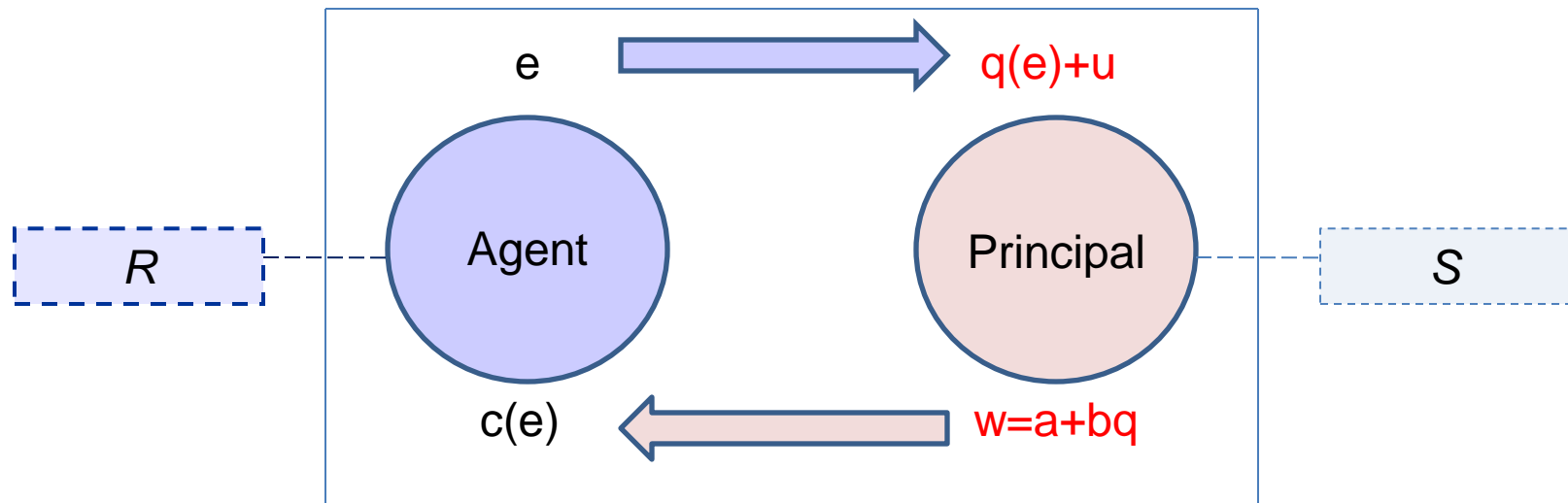


Is this contract efficient?



Objectives for Today

1. Incentives and Insurance with No Hidden Action (review)
2. Moral Hazard: Definition and Implications
3. Incentives and Insurance with Hidden Action
4. Application: Economics of Driving Taxes



Additional Assumptions

- $u \sim (0, \theta)$
- $q(e) = e$
- $c(e) = 0.5e^2$
- A is risk averse
- P is risk neutral
- $R = 0 = S$



The Contract Design

- Choose a contract $[e, a, b]$ to achieve two goals:

1. Max $E[V]$
2. (PC) $E[U] \geq R = 0$

- $E[V] = E[q - w] - 0.5s \text{Var}(q - w)$

$$= \underline{\hspace{10em}}$$

- $E[U] = E[w] - 0.5r \text{Var}(w) - c(e)$

$$= \underline{\hspace{15em}}$$



Optimal Contract

- Use (PC) to substitute for $a+be$ in $E[V]$
- $E[V] = e^{-a - be}$

$$= e^{-(R+0.5e^2+0.5rb^2\theta)}$$

$$= (e^{-0.5e^2}) - 0.5rb^2\theta$$

- First-Order Conditions

(PC) $a^* =$ _____

Summary

- The contract $[e^*, b^*, a^*] = [1, 0, 0.5]$ provides:
 1. Optimal incentives
 - $q'(e^*) = c'(e^*)$
 2. Optimal risk-sharing
 - the risk-neutral principal perfectly insures the risk-averse agent ($b^* = 0$)
- First-best outcome

Application: Contract for Shift Drivers

- If the lease driver is risk neutral, while the shift driver is risk averse, the contract is a salary contract that specifies:
 - Hours of work
 - Fixed payment

- Example:
 - Drive the car five days a week (Monday to Friday), morning shift (7:00AM to 3:00PM) in exchange for a salary of \$500
 - All fares and tips go to the lease driver

Moral Hazard Problem

- Moral Hazard arises when:
 - The principal cannot observe or verify the agent's action
 - The principal and the agent have conflicting interests

- The Moral Hazard exists even if the principal can observe and verify the outcome (but not the agent's action):
 - $q = q(e) + u$
 - Hard to determine what part of outcome is due to agent's action and what part is due to other factors
 - E.g. $8 = x + y$

Main Implication of Moral Hazard

- ⇒ The contract cannot specify the agent's action
(The contract based on e cannot be enforced)
- ⇒ The contract can only specify the payment based on the outcome ($w=a+bq$)
- How does the agent then chooses her action?



Incentive Compatibility Constraint

- Given any contract (a, b) , the agent will choose her action e to maximize her expected payoff
- $\text{Max}_e E[U] = a + be - 0.5rb^2\theta - 0.5e^2$
- First-order condition: _____
- This condition, $\partial E[U]/\partial e = 0$, is known as the incentive compatibility constraint (IC)

Trade-off between risk and incentives

- With symmetric information, we have:
 - $e^* = 1$ Optimal incentives
 - $b^* = 0$ Optimal insurance

 - With asymmetric information, we have from IC:
 - $e = b$
- $\Rightarrow b=1$ $\Rightarrow e=e^*=1$ Optimal incentives, but no insurance!
- $\Rightarrow b^*=0$ $\Rightarrow e=0$ Optimal insurance, but no incentives!



Example: Salary Contract with Hidden Action

- $w=a$

⇒ $E[U] = E[w] - 0.5r\text{Var}[w] - c(e) = a - c(e)$ ($\text{Var}[w]=0$ if $w=a$)

⇒ Optimal action: _____

- $c'(e) > 0$
- $w'(e) = 0$

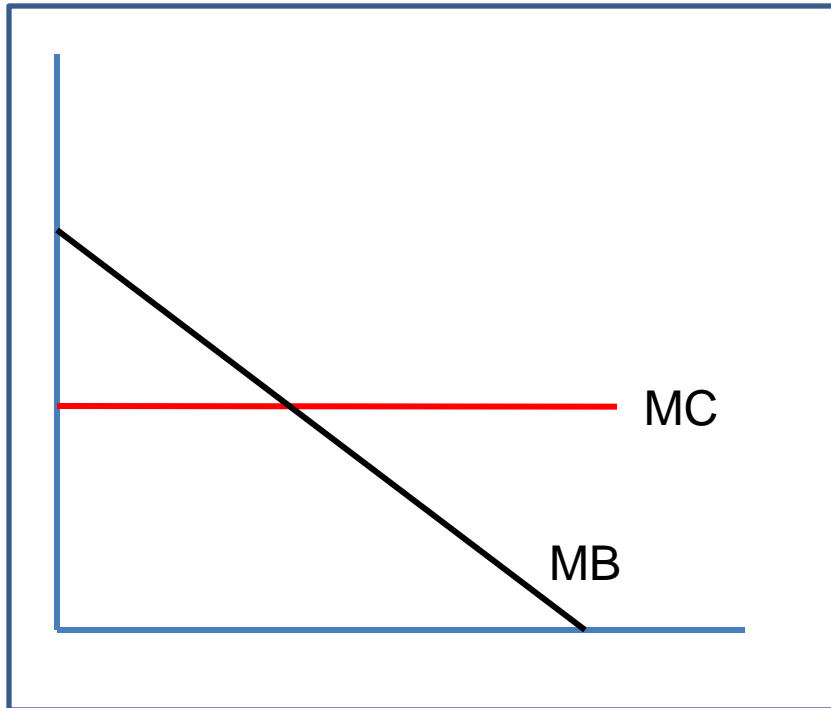
“Well, then, says I, what’s the use you learning to do right when it’s troublesome to do right and ain’t no trouble to do wrong, and the wages is just the same?”

Mark Twain, *Adventures of Huckleberry Finn*

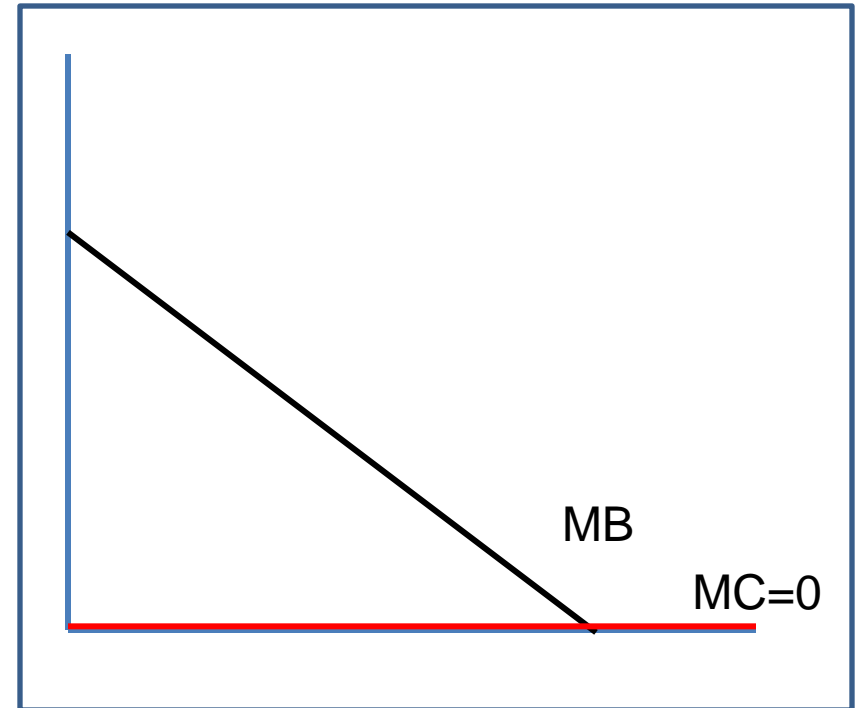
Example: Moral Hazard in Health Care



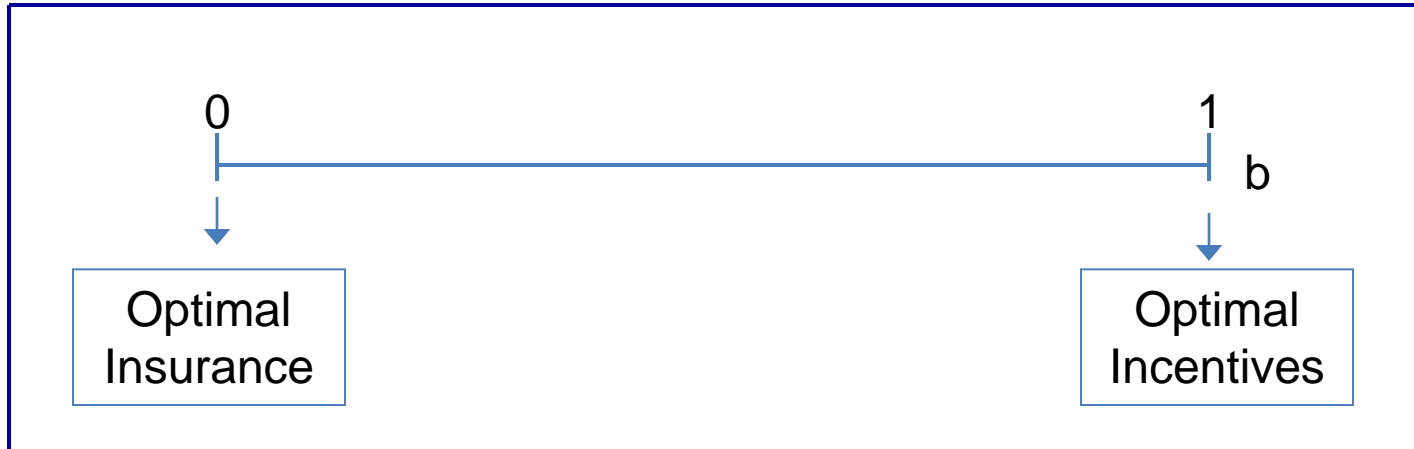
No Insurance



Full Insurance



Role of Parameter $b = w'(q)$

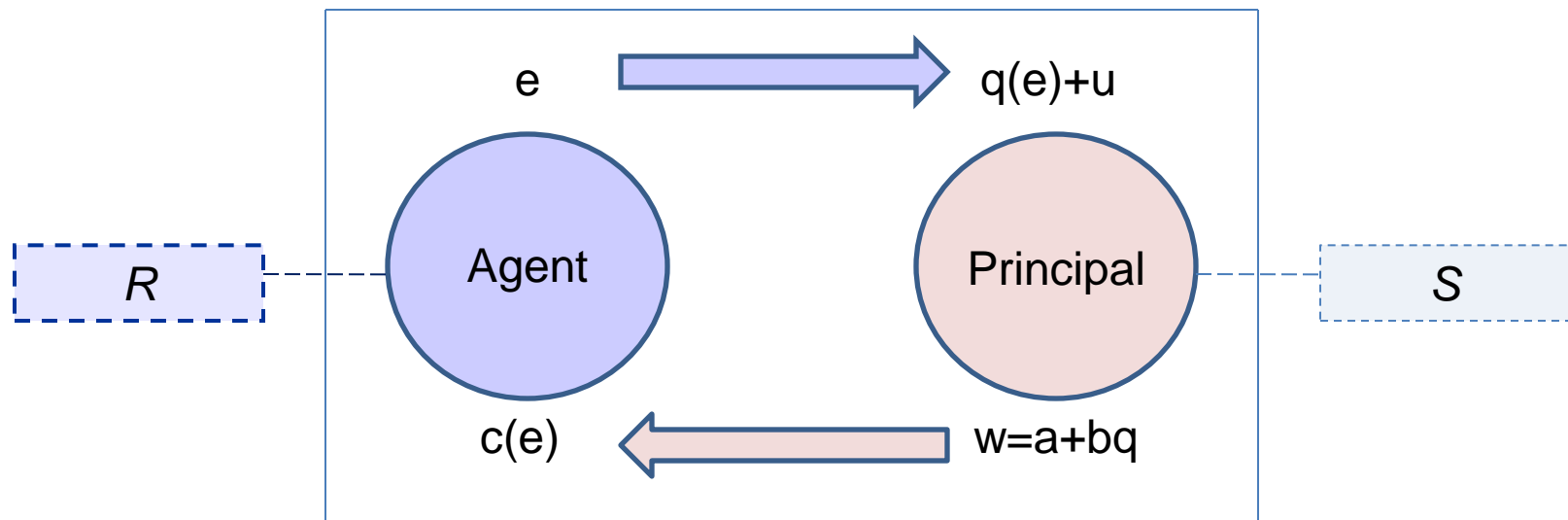


No Hidden Action

- Provide optimal insurance

Hidden Action

- Provide optimal insurance
- Provide optimal incentives



Additional Assumptions

- $u \sim (0, \theta)$
- $q(e) = e$
- $c(e) = 0.5e^2$
- A is risk averse
- P is risk neutral
- $R = 0 = S$
- Agent's action e cannot be observed/verified



The Contract Design

- Choose contract $[a, b]$ such that:



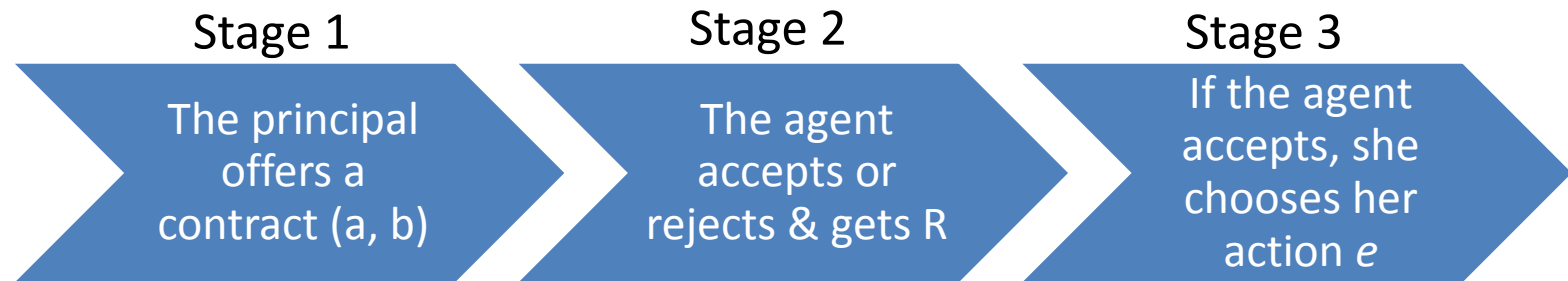
- $$E[V] = E[q-w] - 0.5s\text{Var}(q-w)$$

$$= e - a - be$$

- $$E[U] = E[w] - 0.5r\text{Var}(w) - c(e)$$

$$= a + be - 0.5rb^2\theta - 0.5e^2$$

Timing



Solution by Backward Induction:

- Stage 3: If the agent accepts, what action will he choose?
- Stage 2: Given the action that the agent will choose in Stage 3, when will he accept the contract?
- Stage 1: Given the action that the agent will choose in Stage 3 and given that he will accept the contract in Stage 2, what is the contract that maximizes the principal's objective?

Solution by Backward Induction

Stage 3: Incentive Compatibility Constraint

- For any given contract (a, b) , find the agent's optimal level of effort e
- $\text{Max}_e E[U] = a + be - 0.5rb^2\theta - 0.5e^2$
- (IC) $b - e = 0 \Rightarrow e = b$

Stage 2: Find $a + be$ that satisfies the participation constraint

- $E[U] = a + be - 0.5rb^2\theta - 0.5e^2 = 0$
- \Rightarrow (PC) $a + be = 0.5rb^2\theta + 0.5e^2$

Substitute PC and IC into the principal's objective:

- $E[V] = e - a - be$
- Use PC to substitute for $a + be$:
 - $E[V] = e - (0.5rb^2\theta + 0.5e^2)$
 $= (e - 0.5e^2) - 0.5rb^2\theta$
- Use IC to substitute for $e = b$:
 - $E[V] = (b - 0.5b^2) - 0.5rb^2\theta$



Stage 1: Find b that maximizes the principal's objective

- Max $E[V] = (b - 0.5b^2) - 0.5rb^2\theta$
- FOC: _____

Interpretation

- **(1)** The increase in the expected outcome because higher b induces the agent to provide higher effort. This is beneficial to the principal.
- **(-b)** The increase in the cost of effort because higher b induces the agent to provide higher effort. This is costly to the principal because the agent must be compensated for the additional effort.
- **(-rb θ)** The increase in the risk for the agent because more of his pay is tied to outcome. This is costly to the principal because the agent must be compensated for the additional risk.

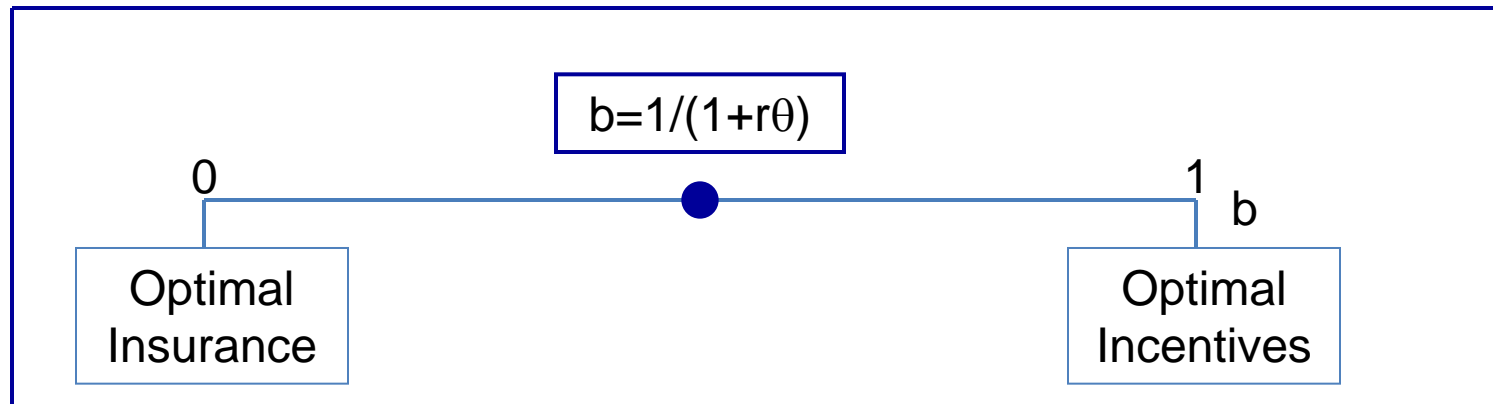


Interpretation – contd.

- FOC implies that _____
- $b > 0$: never provide full insurance if the agent is risk averse
 - with hidden action, optimal to tie pay to performance
- $b < 1$: moral hazard causes efficiency loss
 - the agent provides less effort than if action was not hidden
($e = b < 1 = e^*$)

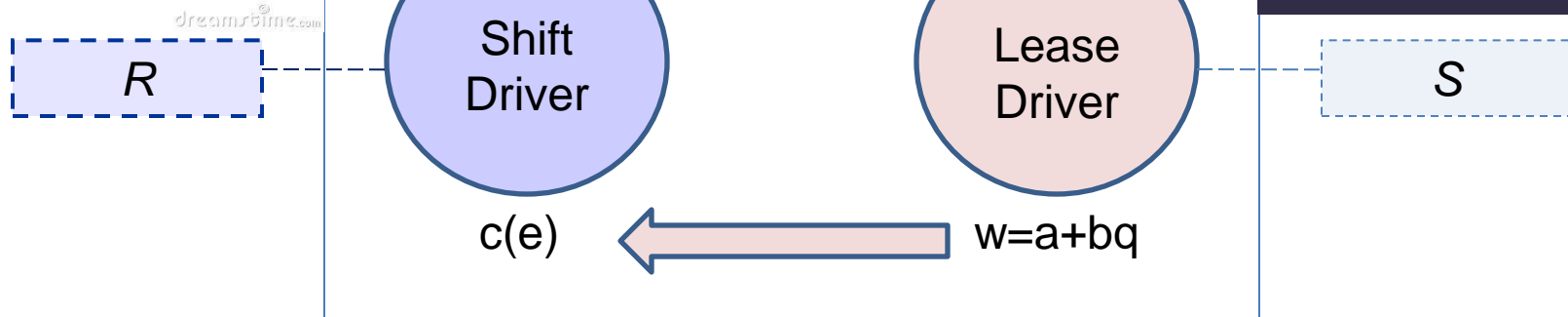
Interpretation – contd.

- **Comparative statics:**
 - Low-power incentives ($b \rightarrow 0$) when θ, r large (risk important)
 - High-power incentives ($b \rightarrow 1$) when θ, r small (risk unimportant)





Elements of the Relationship



- q – the amount of fares and tips
- e – any action that affects the amount of fares and tips, e.g. hours, diligence, friendliness
- u – any other factors that affect fares and tips, e.g. weather, events in the city
- R – working for another lease driver
- S – hiring another shift driver

Optimal Contract: No Hidden Action

Case 1:

The lease driver can observe and verify actions of the shift driver

- Pay a fixed salary for a specified set of actions
- The set of actions balances its impact on the amount of fares and tips and the additional salary that must be paid to the shift driver
- The salary must meet the shift driver's participation constraint.
- Example: pay \$500 for driving the taxi 40 hours per week

Optimal Contract: Hidden Action

Case 2:

The lease driver can't observe and verify actions of the shift driver

- Total compensation consists of two parts:
 - a fixed payment
 - a variable payment that is tied to the total amount of fares and tips
- The share of payment tied to fares and tips is positively related to the predictability of fares and tips and negatively related to the degree to which the shift driver dislikes variation in his pay

- Example: pay \$100 plus 20% of all tips and fares

Main Points

1. **Moral Hazard**: Moral hazard occurs when the principal cannot observe or verify the agent's actions and the principal and agent have conflicting interests.
2. **Implications of Moral Hazard**: When the principal cannot observe or verify the agent's actions, the contract cannot be based on these actions, but can only specify payment that may or may not be tied to the outcome.
3. **Optimal Contract**: When the agent is risk averse, the principal is risk neutral, and the principal cannot observe or verify the agent's actions, it is optimal to tie the agent's pay to the outcome. The extent to which the pay is tied to the outcome depends negatively on the variation in the outcome due to factors that the agent cannot control and on the degree of agent's risk aversion.