Incentives and Insurance

Class 2

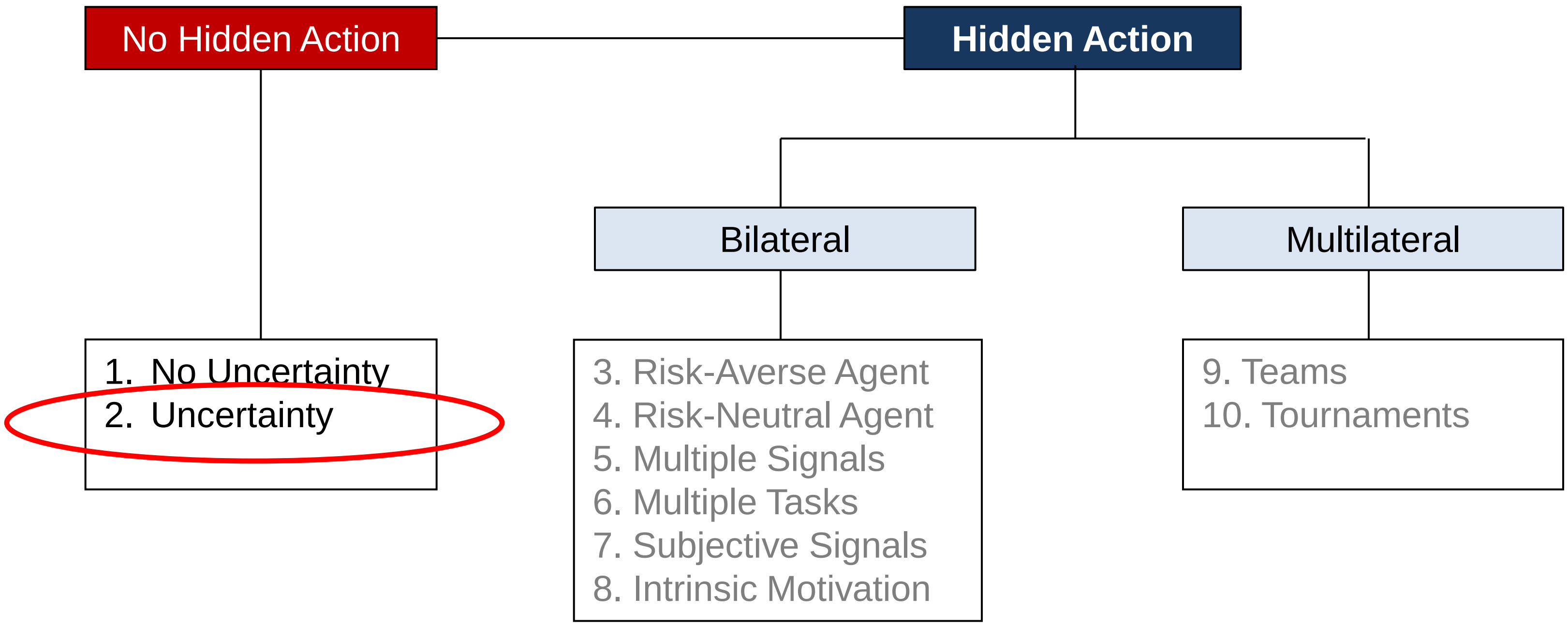
Action	Benefit to the Principal	Cost to the Agent
Dance	$5	$2
Sing	$8	$4

The outside option is $0 for each party.

- As the principal, I hereby declare that I will pay the amount of $____ if the agent dances/sings (circle one).

- As the agent, I hereby accept/reject (circle one).

Big Picture and Road Ahead



Objectives for Today

1. Risk Sharing in Employment Relationships

2. Attitudes Toward Risk

3. Efficient Insurance and Incentive Contract

4. Application: Risk Insurance in Physician Contracts

Uncertainty

- Outcome often depends on factors beside agent's actions

- q=q(e)+u
  - e = factors the agent can control
  - u = factors the agent cannot control (random)

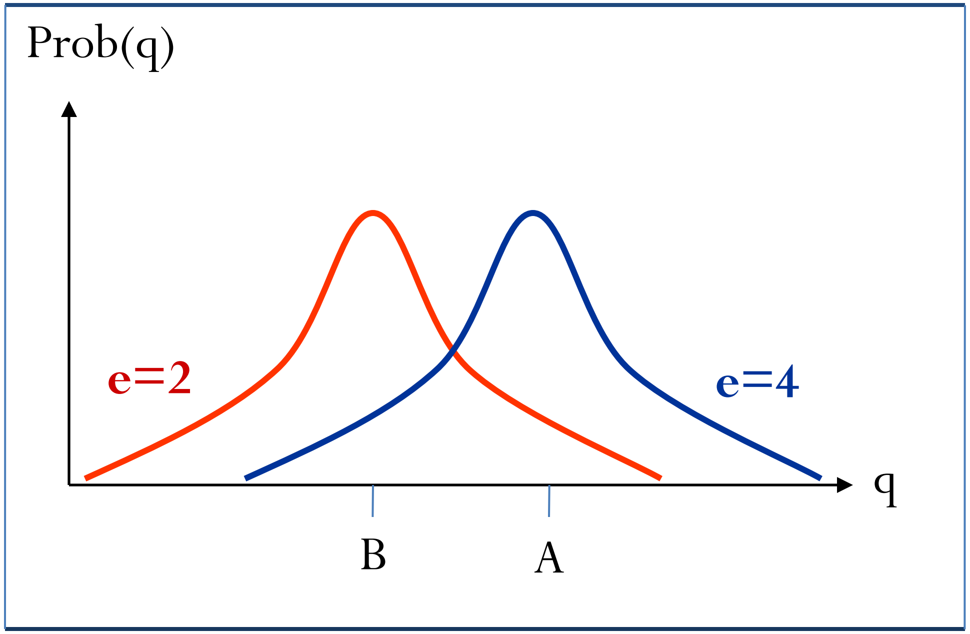
- Suppose E[u]=0, Var[u]=θ

$\Rightarrow$ E[q] = E[q(e)+u] = E[q(e)]+E[u] = __________

$\Rightarrow$ Var[q] = Var[q(e)+u] = Var[u] = __________

Outcome is random

Grades (q) and Hours of Study (e)



$E[q|e=4]=A$ $E[q|e=2]=B$

- Therefore, more likely to get A if e=4.
- Possible to get B even if e=4 or A if e=2!

Your grade is in part random (i.e. not determined only by your actions). This represents **risk** to you.

Risk in Employment Contracts

- $w = a + bq$
  - a = fixed pay
  - bq = variable pay (i.e. depends on the actual outcome)
  - b = $w'(q)$ with $0 \leq b \leq 1$

- Suppose: $q=e+u$, $Var[u]=\theta$

- $Var[w]=Var[a+bq]= Var[a+be+bu]=$______________
- $Var[q-w]=Var[(1-b)q-a]=Var[(1-b)e+(1-b)u-a]=$ ______________

Risk Sharing

- $Var[w]=b^2\theta$ — variation in agent's payoff
- $Var[q-w]=(1-b)^2\theta$ — variation in principal's payoff

- How should this risk be allocated between parties?
  - $b=0$ $\Rightarrow$ Principal bears all risk
  - $b=1$ $\Rightarrow$ Agent bears all risk
  - $0<b<1$ $\Rightarrow$ Principal and agent share risk

When is each option optimal?

Summary of Elements

Element	Description
Parties	Principal, Agent
Production Technology	$q=q(e)+u$ $u \sim (0,\theta)$
Information	e observable and verifiable
Payment	$w = a + bq$
Outside Options	Agent: R Principal: S
Payoffs	Agent: ?? Principal: ??

Question

- When q is random, payoffs are also random
- This represents risk to both parties
- How do parties value this risk?

- With no uncertainty:
  - $V(q-w) = q - w$
  - $U(w)-c(e) = w - c(e)$

How can we reflect that the parties might be concerned about variations in their payoffs when q is random?

Expected Utility

- Let y be a random variable, taking values y_1, y_2,…,y_n
- The probability of each value y_i occurring is p_i

- Let $\pi(y_i)$ be the payoff for a specific value of y_i

- Then, the (von Neumann-Morgenstern) expected utility is:

E[u(y)]= ________________

Example

- Suppose that $\pi(y)=\sqrt{y}$

Outcome (y)	Probability (p)	Payoff (π)	$p\pi$ (y)
4	0.2	2	
9	0.5	3	
16	0.3	4	

- Then, the expected utility $E[\pi(y)]=0.4+1.5+1.2=3.1$.

Convenient Representation of Expected Utility

$$E[\pi(y)] \approx E[y] - 0.5\rho Var[y]$$

- ρ = coefficient of absolute risk aversion (CARA)
  = $-\pi''(y)/\pi'(y)$
  = ‘How much do you dislike risk’
- $0.5\rho Var[y]$ = risk premium
  = ‘How much would you pay to avoid risk?’
- This representation is also called the ‘certainty equivalent’

Example

- Suppose that:
  - $\pi(y)=\sqrt{y}$
  - $E[y]=4$
  - $Var[y]=3$
  - $\rho=2$

- What is the expected value of $\pi(y)$?

- $E[\pi(y)] \approx$ ______________________________

Attitudes Toward Risk

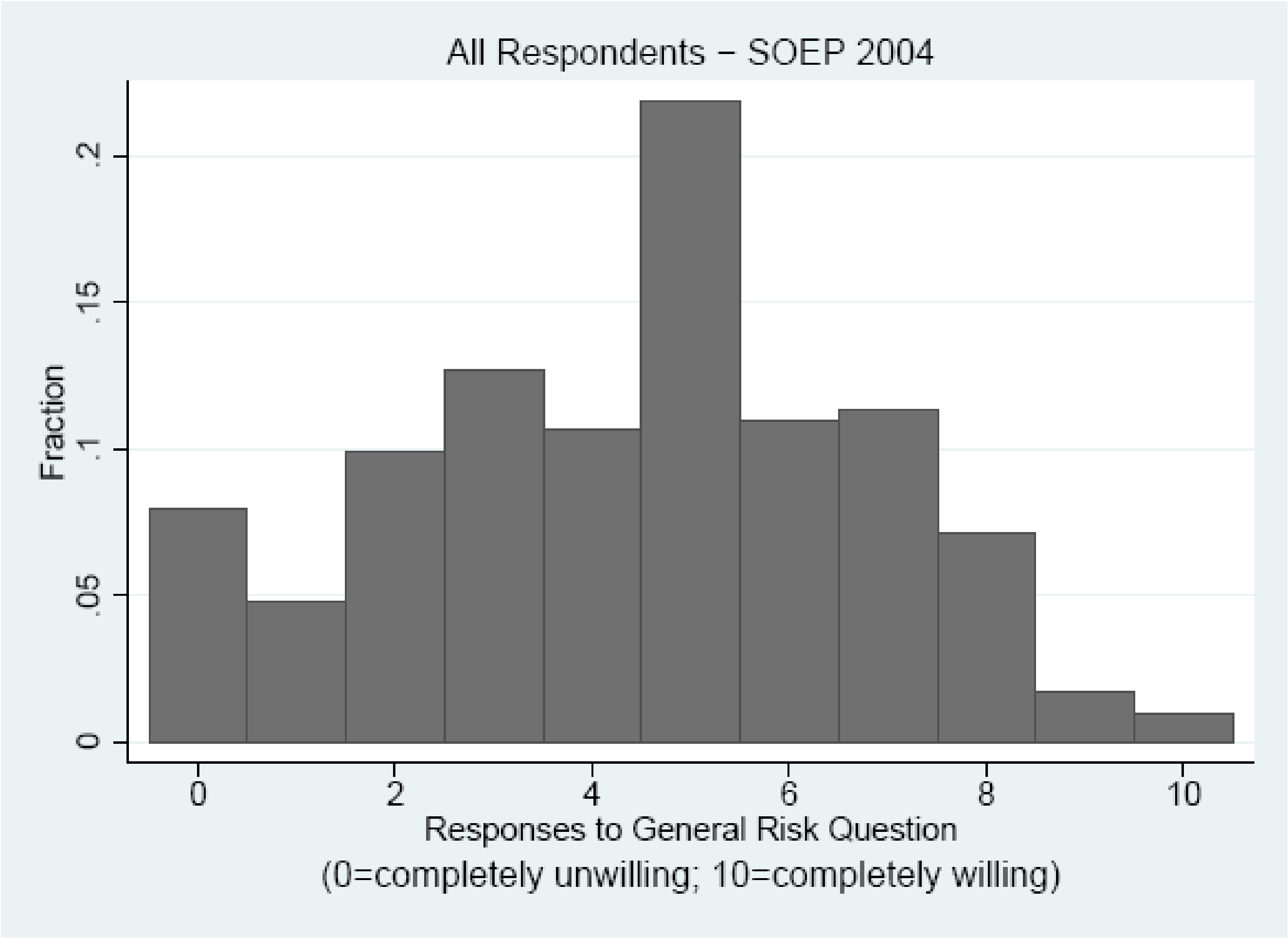
- $E[\pi(y)]=E[y]-0.5\rho Var[y]$

- $\rho > 0 \Leftrightarrow$ 'Dislike' Variance
  $\Leftrightarrow$ 'Willing to pay to reduce Variance' (Risk Premium > 0)
  $\Leftrightarrow$ Risk Averse

- $\rho = 0 \Leftrightarrow$ Don't care about Variance
  $\Leftrightarrow$ 'Not willing to pay to reduce Variance' (Risk Premium = 0)
  $\Leftrightarrow$ Risk Neutral

On a scale of 0 to 10, where 0 is completely unwilling and 10 is completely willing, how willing are you to take risk in general?

Figure 1: Willingness to Take Risks in General



Source: Dohmen et al. (2005), IZA DP No. 1730

Expected Utility - Agent

- Recall that the expected utility for a random variable y using a payoff function $\pi(y)$ can be written as $E[\pi(y)] \approx E[y]-0.5\rho Var[y]$

- For the agent,
  - The random variable is w (i.e. wage, since it depends on q)
  - The payoff function is $U=u(w)-c(e)$
  - Let also the CARA for the agent be denoted by r

➢ $E[u(w)]-c(e) \approx$ ______________________________

= ____

Expected Utility - Principal

- $E[\pi(y)] \approx E[y]-0.5\rho Var[y]$
- For the principal,
  - The random variable is q-w (i.e. profit)
  - The payoff function is V=V(q-w)
  - Let also the CARA for the principal be denoted by s

> ➢ $E[V(q\text{-}w)] \approx$ ______________________________
>
> = ____

Elements

Element	Description
Parties	Principal, Agent
Production Technology	q=q(e)+u u ~ (0,θ)
Information	e observable and verifiable
Payment	w = a + bq
Payoffs	Agent: E[U]=E[w]-0.5rVar[w]-c(e) Principal: E[V]=E[q-w]-0.5sVar[q-w]
Outside Options	Agent: R=0 Principal: S=0

Expected Utility - Agent

- $E[U]=E[w]-0.5rVar[w]-c(e)$
- We can calculate:
  - $E[w] = E[a+bq] = E[a+bq(e)+bu]$
    $= a+bq(e)+bE[u] = a+bq(e)$
  - $Var[w] = Var[a+bq(e)+bu]=Var[bu]=b^2\theta$
- Therefore:
  - **E[U] = ________________________________**

= Expected wage – risk premium – cost of effort

Expected Utility - Principal

- $E[V]=E[q-w]-0.5sVar[q-w]$
- We can calculate:
  - $E[q-w] = E[q-a-bq] = E[(1-b)q-a]$
    $= E[(1-b)q(e) + (1-b)u - a]$
    $= (1-b)q(e)-a$
  - $Var[q-w] = Var[(1-b)q(e) + (1-b)u - a]=Var[(1-b)u]=(1-b)^2\theta$
- Therefore:

E[V] = ___________________________________

= Expected profit – risk premium

The Contract Design

- Choose a contract (e,a,b) to achieve two goals:

1. Max $E[V] = (1-b)q(e)-a - 0.5s(1-b)^2\theta$
2. $E[U] = a+bq(e) - 0.5rb^2\theta - c(e) \geq R = 0$ (PC)

   $\Leftrightarrow a+bq(e) = c(e) + 0.5rb^2\theta$

- Use (PC) to replace a+bq(e) in the objective function

➢ Max E[V] = ___________________________________

- Incentive and insurance problems separable!!

Optimal Incentives

- Choose e to maximize the expected value of relationship
- Max q(e) – c(e)
- First-order condition:
- Same condition as with uncertainty!

Optimal Risk Sharing

- Choose b to minimize the sum of risk premiums

○ Max – $[0.5s(1-b)^2\theta + 0.5rb^2\theta]$

$\Leftrightarrow$ Min $[0.5s(1-b)^2\theta + 0.5rb^2\theta]$

$\Leftrightarrow$ Min Risk Premium^{Principal} + Risk Premium^{Agent}

- First-order condition: $-s(1-b^*)\theta + rb^*\theta = 0$

b* = ______________

- Optimal risk sharing depends on risk preferences.

Risk Sharing: Three Common Cases

- Recall that $b^*=s/(r+s)$
- Case 1: Risk-averse principal, risk-neutral agent
  - $s>0$, $r=0 \Rightarrow$ __________ Agent bears all risk
- Case 2: Risk-neutral principal, risk-averse agent
  - $s=0$, $r>0 \Rightarrow$ __________ Principal bears all risk
- Case 3: Risk-averse principal, risk-averse agent
  - $s>0$, $r>0 \Rightarrow$ ____________ Parties share risk

Application: Physician Payment Contracts

- Let q be the value of medical services provided by physician

- q is likely to be random
  - It depends in part on physician effort (non-random)
  - It also depends on the number of sick patients, the level of care they require, and many other factors (random)

- General type of payment
  - $w = a + bq$
  - Higher b means that physician absorbs more risk

Optimal Insurance Contract

- Case 1: Risk-averse physician, risk-neutral Minister of Health
  - ______________________________
- Case 2: Risk-neutral physician, risk-averse Minister of Health
  - ______________________________
- Case 3: Risk-averse physician, risk-averse Minister of Health
  - ______________________________

Main Points

1. **Twin goals of incentives and risk insurance**. When the outcome of the agent's actions depends in part on factors the agent cannot control, the optimal contract must provide right incentives and optimal risk sharing.
2. **Benefits of Risk Sharing.** In general, the risk of each party can be reduced when it is shared optimally between the principal and the agent.
3. **The optimal type of risk sharing depends on risk preferences.** Specifically, the share of agent's pay tied to the realized outcome is lower the more risk-averse is the agent compared to the principal.