Tournaments

Class 10

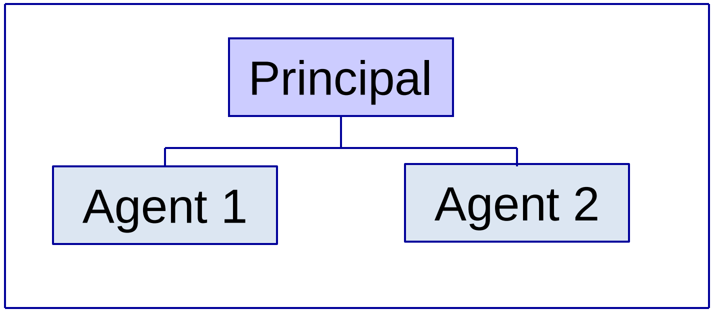
Design for ROM Extension

- The Royal Ontario Museum
- Open competition for the design of its new extension
- Many architects competed
- Architect Daniel Libeskind won
- All other architects lost

Isn’t this wasteful use of other architects’ time and effort?

Teams and Tournaments

- Multilateral contracts
  - We'll consider 'one principal-two agents' contracts



- Two types of contracts:
  - Teams : agents co-operate with each other
  - Tournaments : agents compete against each other (today)

No Hidden Action

1. No Uncertainty
2. Uncertainty

Hidden Action

Bilateral

3. Risk-Averse Agent
4. Risk-Neutral Agent
5. Multiple Signals
6. Multiple Tasks
7. Subjective Signals
8. Intrinsic Motivation

Multilateral

9. Teams
10. Tournaments

Objectives for Today

1. Tournament Model

2. Application: NASCAR

3. Advantages and Disadvantages of Tournaments

Structure of Tournaments

- Two or more agents compete
- Winner obtains higher prize than loser
- Winning depends on agents' effort

$\Rightarrow$ Competition may provide right incentives

Examples

- Promotion in firms
- Academic scholarships (SAT, GRE, etc.)
- Professional sports (hockey, tennis, golf, etc.)

Timing

- Stage 1: Principal announces winning prize W and losing prize w
- Stage 2: Two agents decide whether to participate
- Stage 3: If the agents accept, they choose their actions
- Stage 4: The agent with a higher output wins

Stage 4: Winning Probability

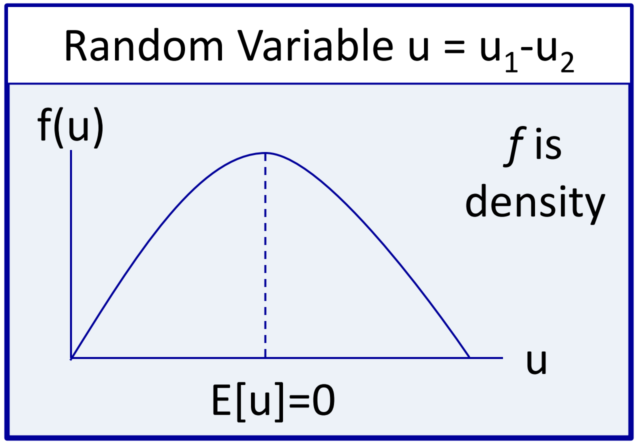
- $q_1 = e_1 + u_1$, $E[u_1]=0$
- $q_2 = e_2 + u_2$, $E[u_2]=0$
- Agent 1 wins if $q_1 > q_2$
- $p = Prob(q_1 > q_2)$

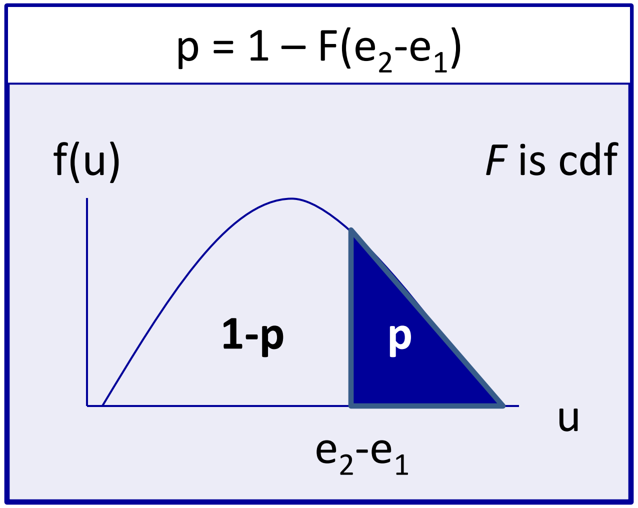
  = ____________________

  = ____________________

  = ____________________

  = ____________________





Stage 3: Choice of Action: Agent 1

- $E[U_1] = pW + (1-p)w - c(e_1)$
  $= w + p(W-w) - 0.5e_1^2$
  = ______________________________

- First-order condition:
  - $(\partial F(e_2-e_1)/\partial e_1)\,(W-w) - e_1 = 0$

    ____

- Recall that $\partial F(x)/\partial x = f(x)$.

Stage 3: Choice of Action: Agent 2

- $E[U_2] = (1-p)W + pw - c(e_2)$

  $= W - p(W-w) - 0.5e_2^2$

  $= W - [1-F(e_2-e_1)](W-w) - 0.5e_2^2$

  $= w + F(e_2-e_1)](W-w) - 0.5e_2^2$

- First-order condition:
  - $(\partial F(e_2-e_1)/\partial e_2)\ (W-w) - e_2 = 0$

$$f(e_2-e_1)(W-w) = e_2$$

Identical agents = Identical actions

- The first-order conditions:
  - $f(e_2 - e_1)(W - w) = e_1$
  - $f(e_2 - e_1)(W - w) = e_2$

⇨ $e_1 = e_2 =$ ______________

- Further, the probability of winning **p =** ______________
  - $p = 1 - F(e_2 - e_1) = 1 - F(0) = 0.5$ since $E[u]=0$

- Therefore, both agents choose identical actions and have identical probability of winning!

Stage 2: Participation Constraints

- $E[U_1] = pW + (1-p)w - c(e_1)$

  $= w + p(W-w) - c(e)$

  $= w + 0.5(W-w) - 0.5e^2$ (recall that p=0.5)

  $= 0.5(W+w) - 0.5e^2 \geq R = 0$

➢ ___________

- This is the same participation constraint for Agent 2 (exercise).

Stage 1: Choice of Prizes

- $e = f(0)(W-w)$ IC
- $W+w = e^2$ PC

- If the principal wants to induce $e^*=1$,
  - $1= f(0)(W-w)$
  - $W+w = 1$

> **$W^* =$ ____________**
>
> **$w^* =$ ____________**

Interpretation and Implications

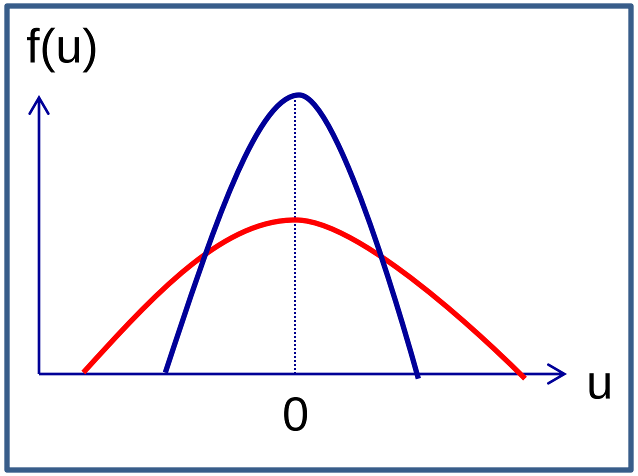
$$e = f(0)(W-w)$$

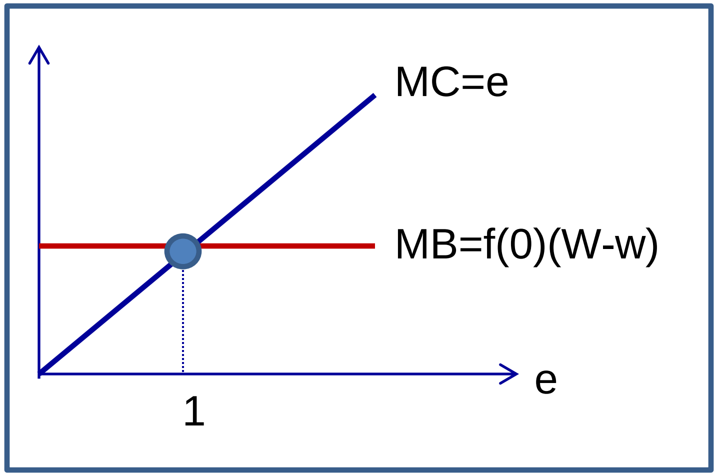
Agents' actions depends positively on f(0)

- f(0) is 'importance of luck'
- Low f(0) means luck is important

Agents' actions depends positively on W-w

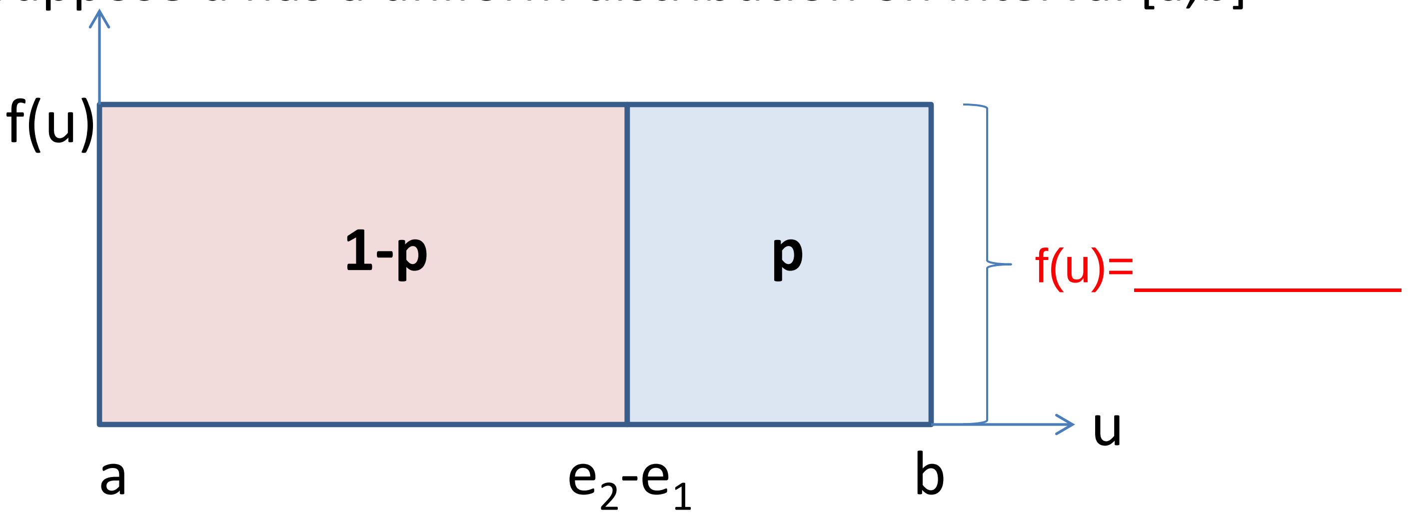
- W-w is the prize spread
- W-w could be set to get e*





Special Case: Uniform Distribution

- Suppose u has a uniform distribution on interval [a,b]



p $= 1 - F(e_2-e_1)$
= ________________
= ________________

Winning Probability with Uniform Distribution

- Suppose:
  - $q_1 = e_1 + u_1$
  - $q_2 = e_2 + u_2$
  - $u_1 - u_2$ is distributed uniformly between -1 and 1

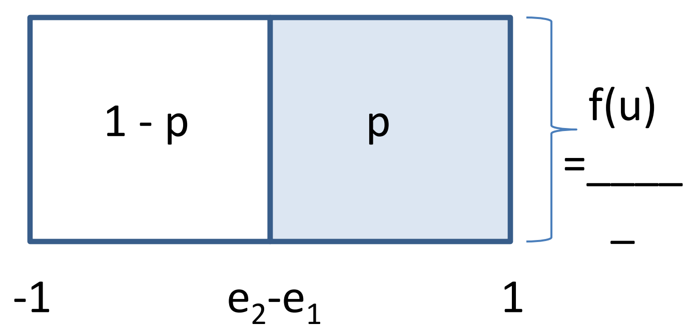
- Then, the probability that agent 1 wins is equal to:

  - p $= Pr(q_1 > q_2)$

    $= Pr(u_1 - u_2 > e_2 - e_1)$

    = ______________

    = ______________



Expected Payoffs with Uniform Distribution

$E[U_1] = pW + (1-p)w - c(e_1)$

$= w + p(W-w) - 0.5e_1^2$

$= w + 0.5(W-w) + 0.5(e_1-e_2)(W-w) - 0.5e_1^2$

$E[U_2] = (1-p)W + pw - c(e_2)$

$= W - p(W-w) - 0.5e_2^2$

$= W - 0.5(W-w) - 0.5(e_1-e_2)(W-w) - 0.5e_2^2$

Application: NASCAR

Becker and Huselid (1992)

Sample

- 28 NASCAR races in 1990
- 44 drivers who competed in at least 5 races

Multivariate Regression Model

$$y_i = \alpha + \beta D_i + \lambda X_i + \varepsilon_i$$

- y = performance measure
- D = prize spread (W-w)
- X = control variables

Definition of Variables

Variable	Type	Definition
Adjusted Finish	Performance (Y)	Finish position, adjusted for speed of the race
Spread	Main independent variable (D, treatment)	Average prize for first n positions relative to last m positions
Start Position	Control (X)	Starting position
Lap Length	Control (X)	Length of lap
Caution Flags	Control (X)	Number of caution laps

Results

Dependent Variable = Adjusted Finish	
Variables	**Estimate (t-stat)**
Spread	**-1.1187 (5.69)**
Caution Flag	0.1676 (16.13)
Start Position	0.2247 (3.20)
Lap Length	0.7742 (0.79)

Advantages of Tournaments

- Improve incentives for all agents in tournament.
- Serve a complementary function of ranking agents.
- Lower measurement costs (relative, not absolute).
- May filter out risk common to all agents.

  $q_1+\varepsilon > q_2+\varepsilon \leftrightarrow q_1 > q_2$ (ε = common risk)

- Useful with frequent technological changes.

  $Aq_1 > Aq_2 \leftrightarrow q_1 > q_2$ (A = technology)

Disadvantages of Tournaments

- May induce agents to sabotage each other.
- May induce agents to collude against the principal.
- Agents may select which tournament to participate.
- No incentives to co-operate with other agents.

Main Points

1. **Tournaments**: Competition between agents can provide powerful incentives. In general, the agents' actions are higher the higher the gain from winning and the more important are their actions relative to luck in determining the winner.
2. **Advantages and Disadvantages of Tournaments**: Competition between agents can induce the efficient outcome. Further, this payment method reduces measurement costs, filters out common risks, and is robust to technological changes. However, tournaments discourage cooperation and participation by disadvantaged groups, which may limit their use in practice.