

Basic Incentive Contract

Class 1

Apply By: Tuesday, March 26, 2013 11:59 pm EDT

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ECONOMIST

Organization: Ministry of Infrastructure
Division: Infrastructure Policy and Planning Division
City: Toronto
Job Term: 1 Permanent
Job Code: 17PEC - Ecomcs Stat Ama Unit
PEC17
Salary: \$65,898.00 - \$79,912.00 per year
Posting Status: Open
Job ID: 49862



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The Infrastructure Policy Branch is looking for an Economist to contribute to the development and implementation of provincial infrastructure policies and programs related to health, social and justice infrastructure.

What can I expect to do in this role?

In this role, you will:

- plan and conduct complex policy, statistical and socio-economic research and financial analysis
- provide strategic policy analysis, advice and expertise regarding health, social and justice infrastructure
- manipulate and compile data using qualitative and quantitative analysis
- identify and analyze contentious issues and provide options and recommendations to senior management
- prepare and present research findings, policy options, briefings to senior executives
- establish a network of internal and external contacts and manage effective working relationships with stakeholders

Employment Contracts

- Two main elements in employment contracts:
 1. Duties and responsibilities ('roles')
 2. Compensation ('salary')



How are these elements determined?

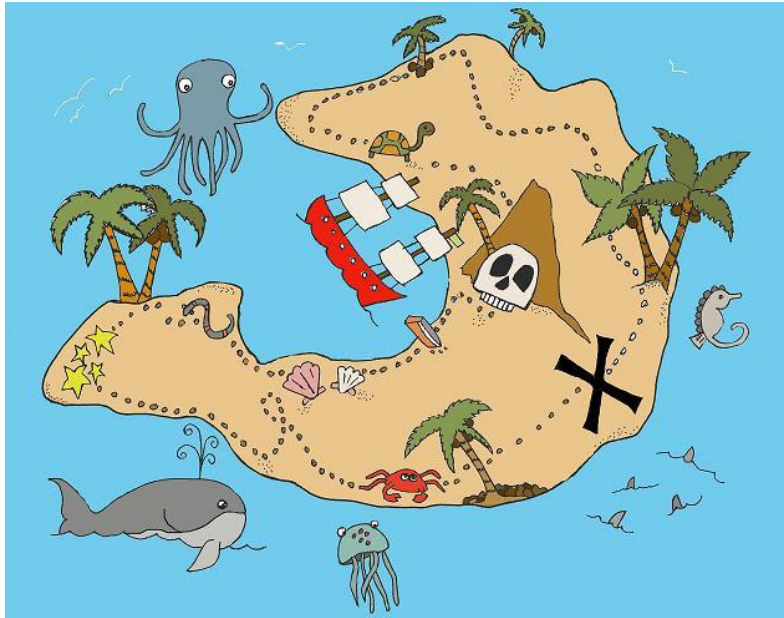


Is there a best way to design an employment contract?

Economist's Perspective

- Employment is a voluntary relationship
 - Both parties will enter the relationship only if they gain

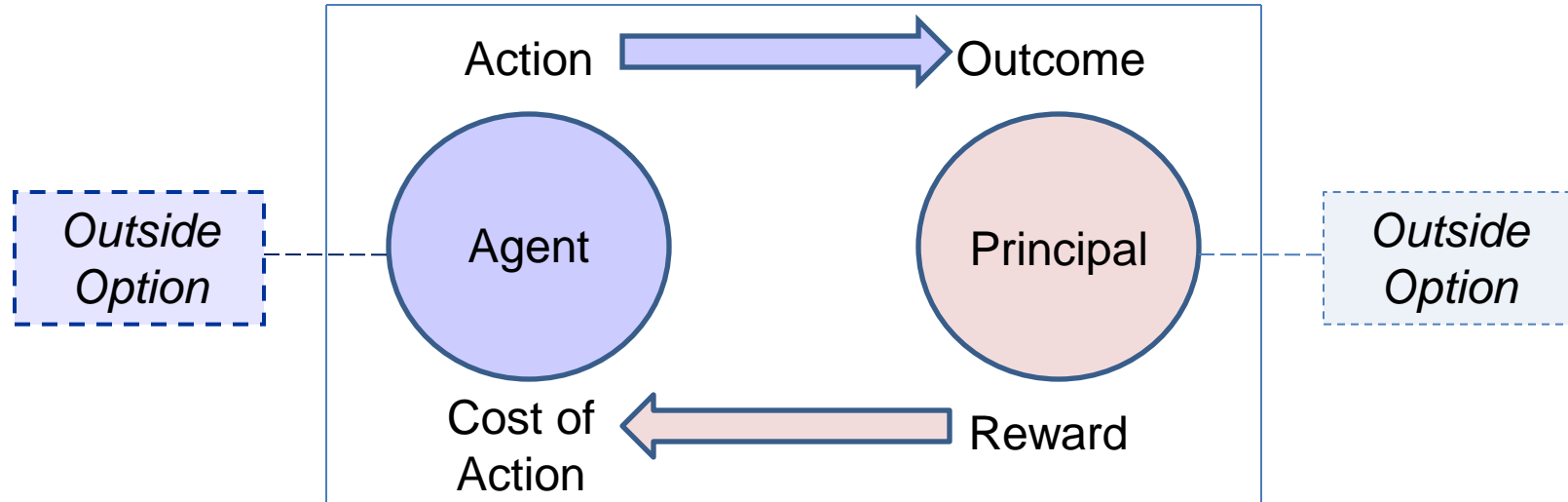
- Two main questions:
 - How to maximize this gain? (Efficiency)
 - How to divide this gain? (Equity)



Objectives for Today

1. Describe Elements of Employment Relationship
2. Find an Efficient Employment Contract
3. Equity in Employment Contracts

Overview



- Two parties: the principal and the agent
- The agent takes an action (i.e. duties and responsibilities)
- The action affects an outcome the principal cares about (e.g. profits)
- The action is costly to the agent (e.g. value of leisure)
- The principal rewards the agent for his action (i.e. compensation package)
- Each party can opt out and receive their outside option

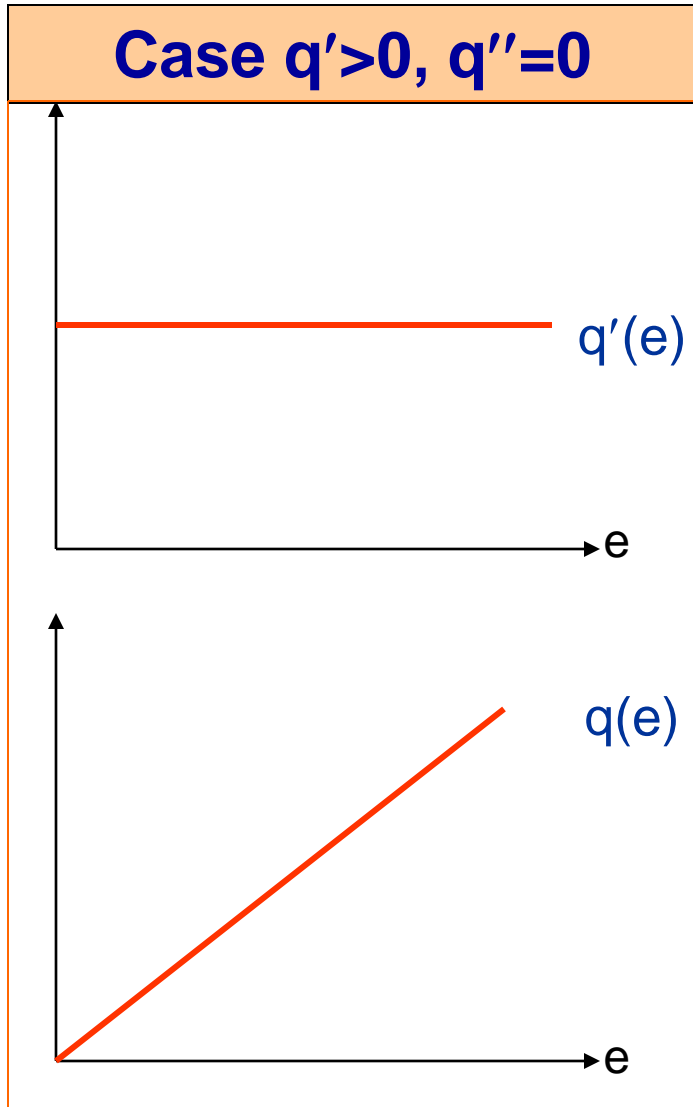
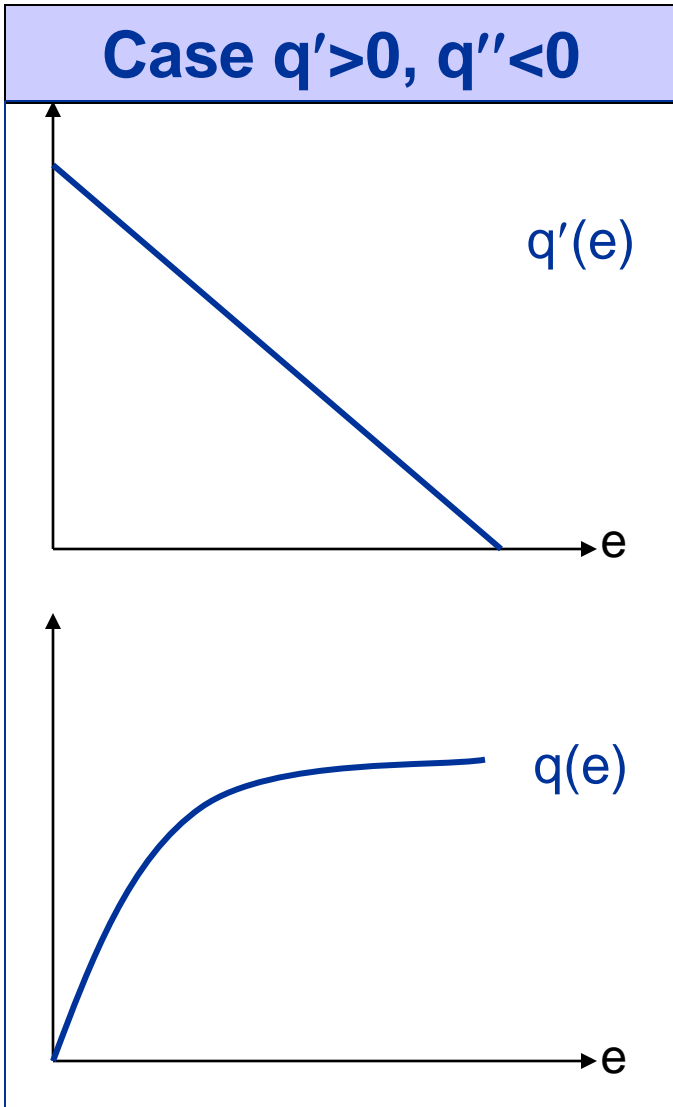
Agent's Action

- $e \geq 0$ (e.g. hours of work)
- $e = 0$ is the agent's privately optimal action
- e is usually interpreted as 'effort'

- In this class, we will assume that the agent's action can be:
 - observed by the principal
 - verified in the court of law

Outcome

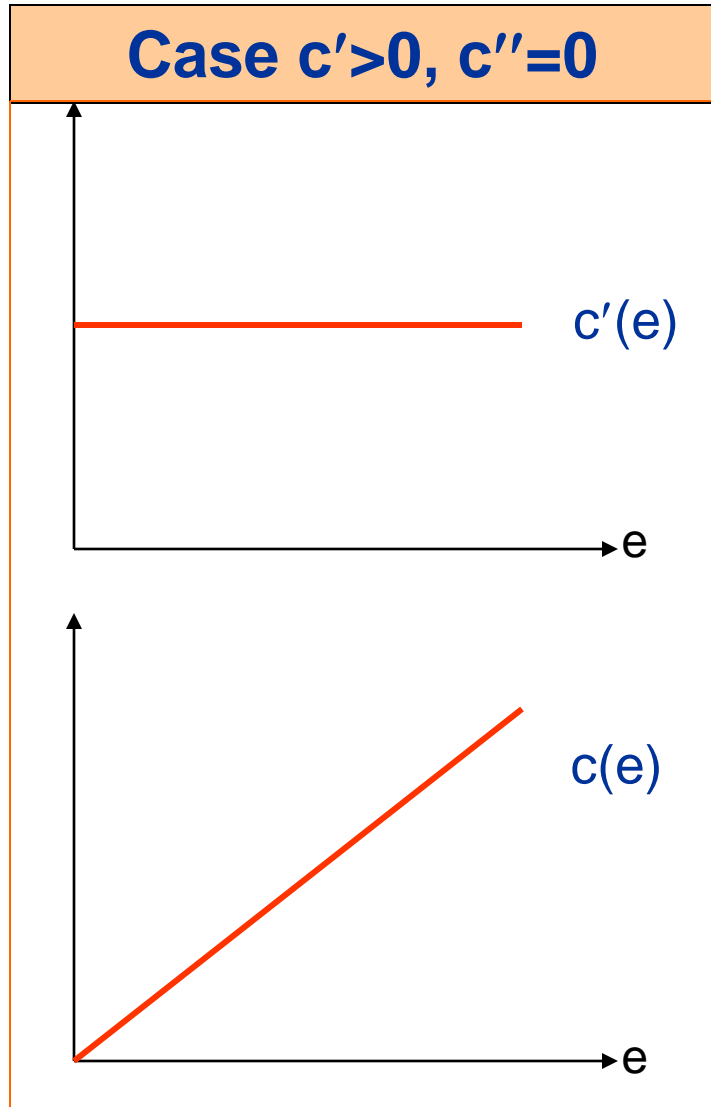
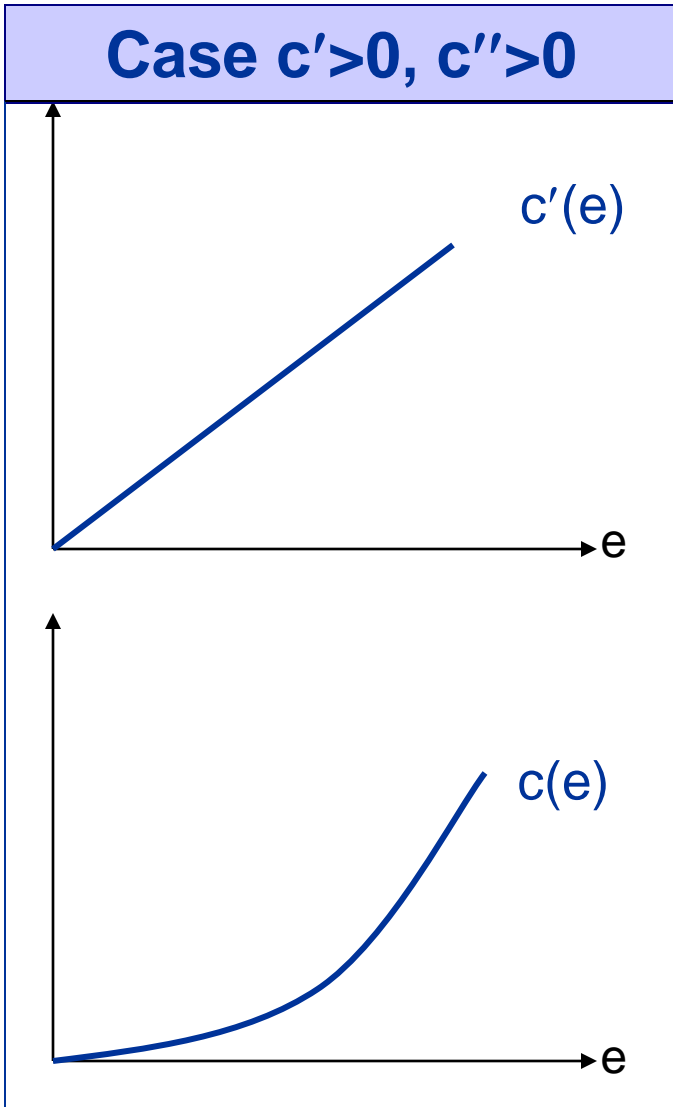
- q is interpreted as the value of outcome (e.g. profits)
- $q(e)$ is the production function
 - $q'(e) > 0$: marginal product
 - $q''(e) \leq 0$: non-increasing marginal product
- In this class, we'll assume that q is:
 - Completely determined by e (no uncertainty)



If $q'' = 0$, is it better to study for 8 hours in 1 day or 8 hours over 2 days?

Cost of Action

- $c(e)$ is the cost of effort function (e.g. disutility of effort)
- $c'(e) > 0$: marginal cost of effort
- $c''(e) \geq 0$: non-decreasing marginal cost



If $c'' > 0$, is it better to study for 8 hours one day or for 4 hours for 2 days?

Agent's Reward and Payoffs

- Let w be the agent's reward.

Principal's Payoff

- $V = q(e) - w$

Agent's Payoff

- $U = w - c(e)$

Outside Options

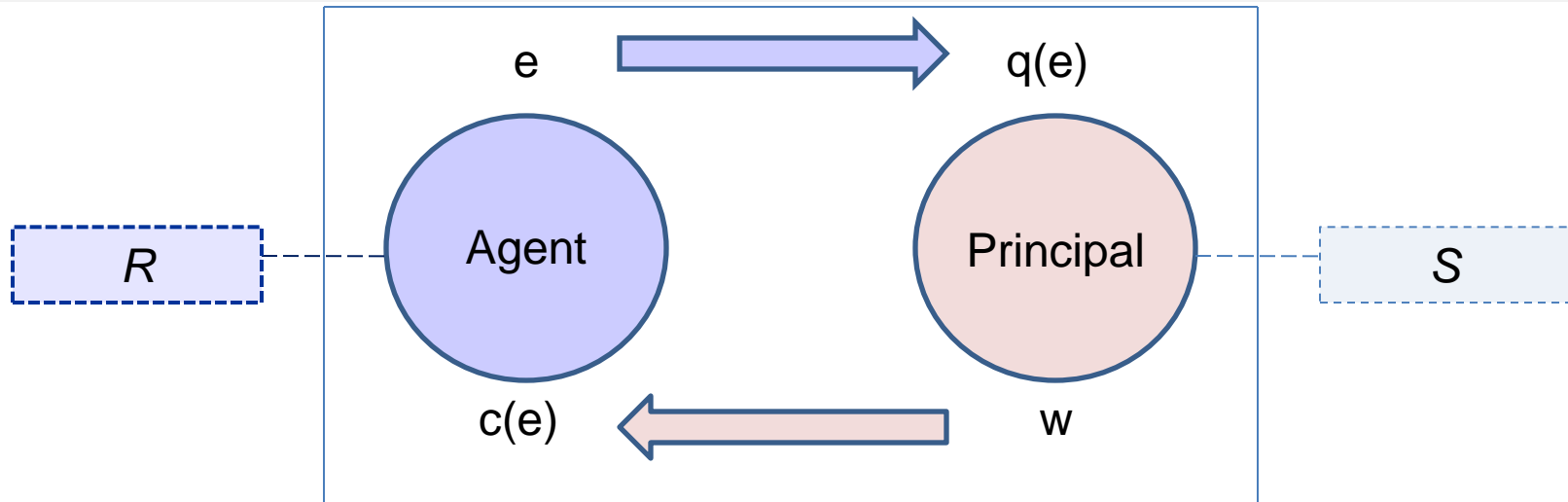
Principal

- S
- Opportunity cost (e.g. hire another agent)

Agent

- R
- Opportunity cost (e.g. alternative employment)

Summary of Elements



Agent's Payoff

- $U = w - c(e)$

Principal's Payoff

- $V = q(e) - w$

Summary of Elements

Element	Description
Parties	Principal, Agent
Production Technology	$q = q(e)$, with $q' > 0$, $q'' \leq 0$ No uncertainty
Information	e observable and verifiable
Payoffs	Agent: $U = w - c(e)$, with $c' > 0$, $c'' \geq 0$ Principal: $V = q(e) - w$
Outside Options	Agent: R Principal: S

Contract between Economist and Ministry of Infrastructure

- Who is the principal? Who is the agent?
- What is the outcome?
- What are the agent's actions?
- How does the outcome depends on actions?
- How do actions impact the agent's utility?
- What is the payoff for each party?
- What is the outside option for each party?



Conflict of Interest

- Contract: $[e, w]$
- A contract that 'best' serves interests of both parties

- The interest of each party:
 - Agent : $\text{Max } U = w - c(e)$
 - Principal : $\text{Max } V = q(e) - w$

- Conflict of interest
 - Agent prefers higher wage, lower effort
 - Principal prefers lower wage, higher effort

Efficiency Principle

- Focus on (Pareto) efficient contracts:
 - Contracts universally accepted by both parties

- Three alternative definitions:
 1. Max $V + U$

 2. Max V subject to $U \geq R$

 3. Max U subject to $V \geq S$



Social Surplus Function

$$SS = V + U$$

$$=$$
$$=$$

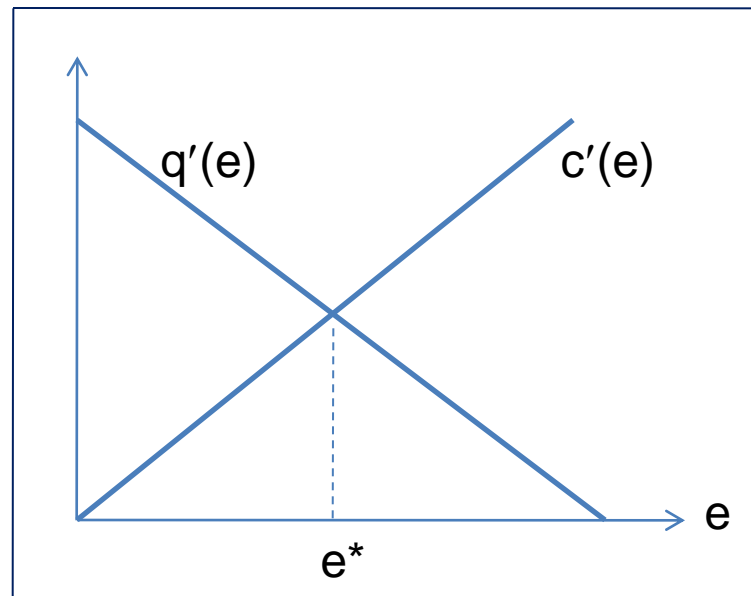
- Captures all *real* costs and benefits
- Independent of transfers (w) between parties
- $q(e)-c(e)$ is the joint value of relationship
- $S+R$ is the joint value of outside options



Efficiency Conditions

1. Choose effort to maximize the value of relationship

- $\text{Max}_e q(e) - c(e)$
- First-order condition:
- Interpretation: $\text{MB}(e^*) = \text{MC}(e^*)$



Efficiency Conditions

2. Compare the value of relationship to outside options

- $q(e^*) - c(e^*) \geq S + R$

⇒ efficient to form relationship

- $q(e^*) - c(e^*) < S + R$

⇒ not efficient to form relationship

Application

- We wish to design a contract for a physician.
- Need information on:
 - shape of $q(e)$ and $c(e)$; R , S
- Suppose:
 - e represents days per week
 - $q(e) = \$600e$
 - E.g. 20 patients per day, \$30 per patient
 - $c(e) = \$60e^2$
 - $R = \$1,000/\text{week}$
 - $S = \$0/\text{week}$





Application...contd.

1. $q'(e^*)=c'(e^*)$

○ $q(e)=$ $\Rightarrow q'(e)=$

○ $c(e)=$ $\Rightarrow c'(e)=$

\Rightarrow

2. $q(e^*)-c(e^*) \geq R + S$??

○ $q(e^*)-c(e^*) =$

○ $R+S=$

\Rightarrow

Participation Constraints

- To be acceptable to both parties, the reward w^* must satisfy two conditions known as **participation** constraints:

- $U = w^* - c(e^*) \geq R$

- $w^* \geq R + c(e^*)$

Agent willing to participate

- $V = q(e^*) - w^* \geq S$

- $w^* \leq q(e^*) - S$

Principal willing to participate

Equity – Division of Gain

- Participation constraints impose upper and lower limit on w^* :
 - $R+c(e^*) \leq w^* \leq q(e^*)-S$
- The level of w^* determines how the gain from the employment relationship is divided between parties – i.e. **equity**

- $w^*=R+c(e^*)$

- $U = w - c(e^*) = R$

- $V = q(e^*)-w^*$
 $= q(e^*)-c(e^*)-R \geq S$

- Agent indifferent between this and outside option
- Principal strictly better off

- $w^*=q(e^*)-S$

- $U = w - c(e^*)$
 $= q(e^*)-c(e^*) -S \geq R$

- $V = q(e^*)-w^* = S$

- Agent strictly better off
- Principal indifferent between this and outside option

Bargaining Power

- Division of gain depends on the bargaining power of each party
- In this course, we'll consider a simple bargaining game:
 - "Take-It-or-Leave-It" offer
 - First-mover advantage
 - Party designing the contract extracts all gain





First-Mover advantage

- How to split \$10 you found on the street with your ‘friend’
- If you cannot agree, it goes to Lost and Found
- You both care only about \$ (i.e. no envy, fairness)

Offer (You, your friend)	You (Accept/Reject)	Your Friend (Accept/Reject)
(\$5,\$5)		
(\$10, \$0)		
(\$0,\$10)		



Which offer would you propose?

Which offer would your friend propose?



Principal Designs Contract

- Max $V=q(e)-w$
subject to: (PC) $U=w-c(e) \geq R \Rightarrow w=$

- Use (PC) to replace w in the principal's objective function
 - Max $V=q(e)-w = q(e)-c(e)-R$
 - First-order condition for e :

 - Given e^* , (PC) then determines w^* :



Agent Designs Contract

- Max $U = w - c(e)$
subject to: (PC) $V = q(e) - w \geq S \Rightarrow$

- Use (PC) to replace w in the agent's objective function
 - Max $U = q(e) - c(e) - S$
 - First-order condition for e :

 - Given e^* , (PC) then determines w^* :
 - $w^* =$

Designing Contracts - Summary

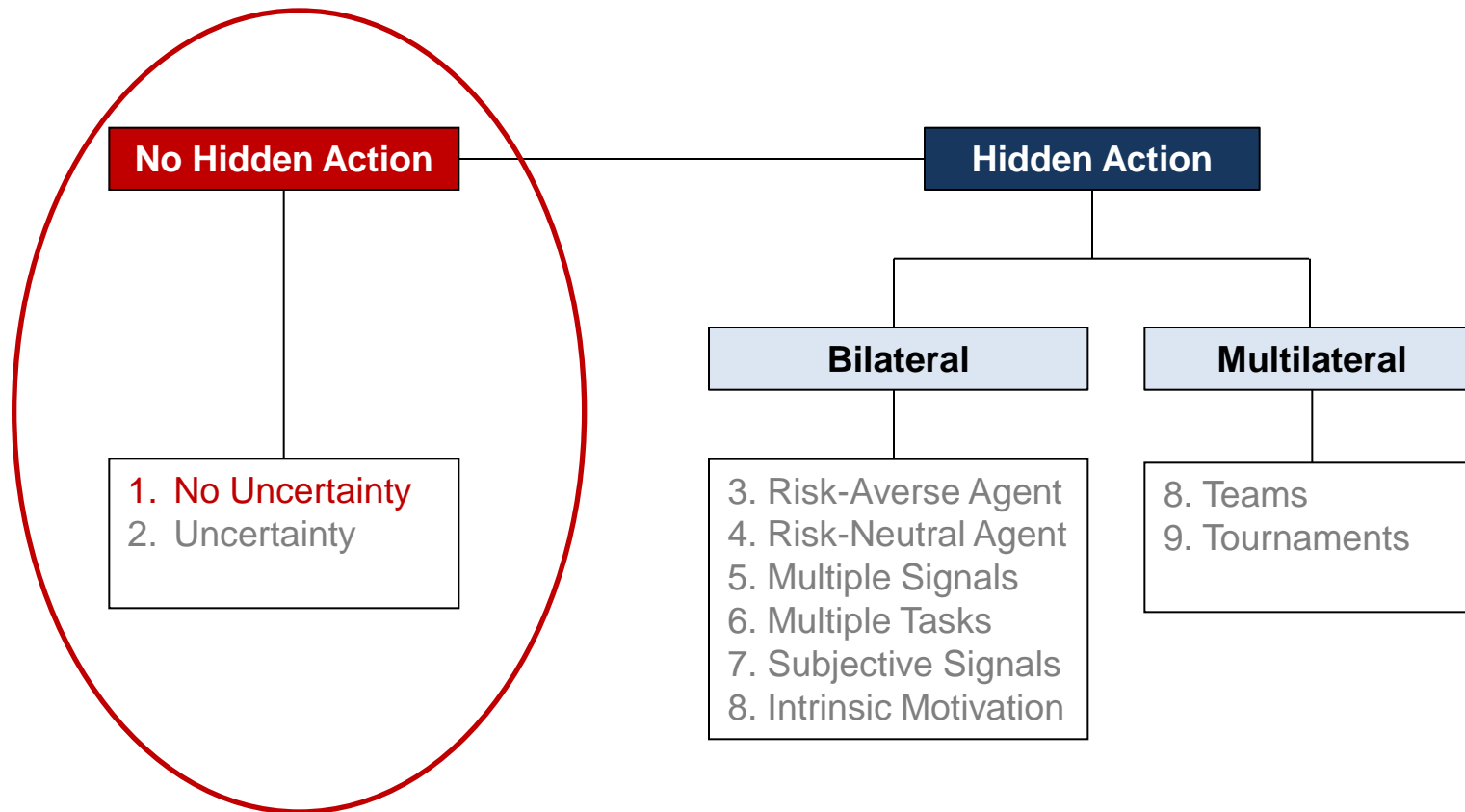
	Social Surplus	Principal Designs Contract	Agent Designs Contract
Objective Function	Max $q(e) - c(e)$	Max $V = q(e) - w$ (PC) $U = w - c(e) = R$	Max $U = w - c(e)$ (PC) $V = q(e) - w = S$
e^*	$q'(e^*) = c'(e^*)$	$q'(e^*) = c'(e^*)$	$q'(e^*) = c'(e^*)$
w^*	$q(e^*) - S \leq w^* \leq R + c(e^*)$	$w^* = R + c(e^*)$	$w^* = q(e^*) - S$

All
Roads
Lead to
ROMA

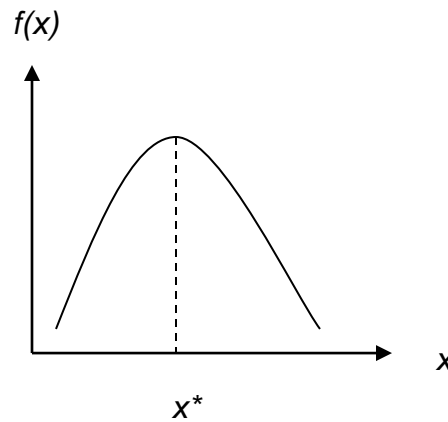


Main Points

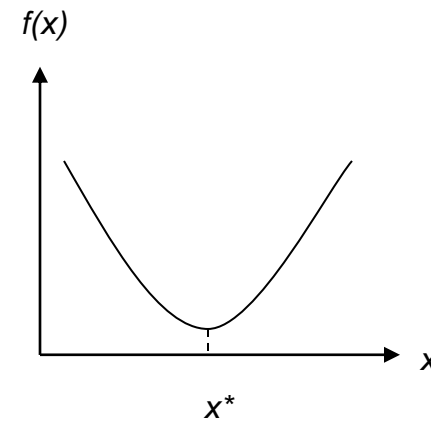
1. Employment relationships can often be conveniently described using the **Principal-Agent Framework**. The main elements include: the parties, the agent's action, the outcome, the cost of action, the payoff of each party, and the outside option for each party.
2. **(Pareto) Efficient Contract**: An efficient contract is defined as a contract that maximizes the payoff of at least one party without making the other party worse off.
3. **Efficiency and MB=MC Principle**: To maximize the gain from the relationship, both benefits and costs must be considered. Technically, the actions that maximize the gain from the relationship equate their incremental costs and benefits.
4. **Not all relationships should be formed**: To determine whether a relationship should be formed, the maximum gain from the relationship should be compared to outside options.
5. **Equity**: As long as it is efficient to form the relationship, there exists a compensation package that is acceptable to both parties. The exact level of compensation will in general depend on each party's bargaining power.



Single Variable Optimization



Maximum



Minimum

First-order condition

$$f'(x^*) = 0$$

$$f'(x^*) = 0$$

Second-order condition

$$f''(x^*) \leq 0$$

$$f''(x^*) \geq 0$$