Homework 9
Efficiency Wages and Deferred Compensation

Problems

A. Efficiency Wages

(1)* You wish to hire a home renovator. The quality of the renovation work is given by q=10e+u, where e is the renovator's effort and u is a random variable with a mean of zero. The cost of effort is e^2. If the renovator shirks (provides e<e*), you can detect this with probability 0.5. What wage contract should you offer to the renovator if you want to induce the efficient level of effort?

(2)* A dentist wants to design an efficiency wage contract for her assistant. The output of the assistant is q=8e+u, where e is the assistant's effort and u is a random variable with a mean of zero. The cost of effort is $e^2/5$. If the assistant shirks (provides e<e*), the dentist can detect this with probability 4/5. If the assistant's outside option is 10, what rent will the assistant enjoy if the dentist designs this contract?

(3)* Henry Ford wishes to design an efficiency wage contract for his workers. The output of each worker is q=6e+u, where e is effort and u is a random variable with a mean of zero. The cost of effort is $e^2/4$. What is the minimum probability of detecting shirking that will make this contract feasible for Henry?

(4)* A store owner pays an efficiency wage to his employees. The output of each employee is q=e+u, where e is the employee's effort and u is a random variable with a mean of zero. The cost of effort is $0.5e^2$. The probability of detecting shirking is p=1/2. The store owner can invest in improving the monitoring technology to increase p to 4/5. How much would the owner be willing to invest in this?

(5)** Consider a one-period employment relationship with the following sequence of events. First, the employer pays w to the worker. Second, the worker supplies effort e. Third, the employer monitors the worker's effort. Fourth, the employer offers a retirement bonus b to workers who are not caught shirking during their period of employment. Effort e can take two values, 0 or 1, and imposes cost 0.25e on the worker. The monitoring technology allows the employer to detect shirking (when e=0) with probability p=0.5. Each unit of effort produces one unit of output. The worker's outside option R is 0.85 and the employer's outside option is 0. Explain what contract [w, b] will induce the efficient outcome in this relationship.

B. Deferred Compensation

(6)* Suppose that the employment relationship lasts for two periods. In each period, the employee's productivity is q=6e+u, where e is effort and u is a random variable with a mean of zero. The cost of effort is e^2 in each period. The employer can detect shirking with the probability of 0.5. What is the optimal wage profile that would induce the worker to provide the efficient level of effort in each period? Assume that the employee does not discount future income.

(7)* Consider the employment relationship described in Problem 6, but now suppose that the productivity is q=6e+u in the first period and q=4e+u in the second period. Similarly, the cost of effort is now $0.5e^2$ in the first period and e^2 in the second period. What is the optimal wage profile now?

(8)* Consider the employment relationship described in Problem 6, but now assume that the employee discounts the future benefits using a discount factor of 0.9. What is the optimal wage profile in this case? How does the introduction of discounting affect the first-period wage and why?

(9)* Explain why a wage profile with w_1>…>E[q]>…>w_N is unlikely to induce optimal incentives when the worker's productivity E[q] is constant over time.

(10)* Based on Gunderson and Pesando (1999), discuss main arguments for and against the mandatory retirement.

Solutions

(1) The efficient level of effort satisfies the first-order condition $E[q'(e^*)]=c'(e^*)$, or $10=2e^*$, from which it follows that $e^*=5$. If the renovator shirks, his expected utility is $0.5*0+0.5w-c(e)=0.5w$ since it is optimal to provide no effort if he shirks. If the renovator does not shirk, his expected utility is $w-c(e^*)=w-5^2$. Therefore, the no shirking condition is $w-5^2\geq 0.5w$, which implies that the wage contract specifies $w=50$ if $e\geq 5$ and $w=0$ otherwise.

(2) The efficient level of effort satisfies the first-order condition $E[q'(e^*)]=c'(e^*)$, or $8=2e^*/5$, from which it follows that $e^*=20$. If the assistant shirks, his expected utility is $(4/5)*0+(1/5)w-c(e)=w/5$. If the assistant does not shirk, his expected utility is $w-c(e^*)=w-20^2/5$. Therefore, the no shirking condition is $w-20^2/5\geq w/5$, which implies that the wage contract specifies $w=100$ if $e\geq 20$ and $w=0$ otherwise. The assistant's expected utility is then $100-20^2/5=20$, which is greater than $R=10$ by 10. Therefore, the rent is 10.

(3) The efficient level of effort satisfies the first-order condition $E[q'(e^*)]=c'(e^*)$, or $6=2e^*/4$, from which it follows that $e^*=12$. If the worker shirks, his expected utility is $p*0+(1-p)w-c(e)=(1-p)w$ since it is optimal to provide no effort if he shirks. If the worker does not shirk, his expected utility is $w-c(e^*)=w-12^2/4$. Therefore, the no shirking condition is $w-12^2/4\geq(1-p)w$, which implies that the wage contract specifies $w=36/p$ if $e\geq 12$ and $w=0$ otherwise. The expected profit is then $E[q]-w=6(12)-36/p$. This profit is positive if $p\geq 1/2$.

(4) The efficient level of effort satisfies the first-order condition $E[q'(e^*)]=c'(e^*)$, or $1=e^*$. If the employee shirks, his expected utility is $(½)*0+(½)w-c(e)=w/2$ since it is optimal to provide no effort if he shirks. If the employee does not shirk, his expected utility is $w-c(e^*)=w-0.5*1^2$. Therefore, the no shirking condition is $w-0.5\geq w/2$, which implies that the wage contract specifies $w=1$ if $e\geq 1$ and $w=0$ otherwise. The expected profit is then $E[q]-w=1-1=0$. With $p=4/5$, the no shirking condition becomes $w-0.5\geq w/5$, from which it follows that the wage contract would specify $w=5/8$ if $e\geq 1$ and $w=0$ otherwise. The expected profit is then $1-5/8=3/8$. Therefore, the store owner is willing to invest up to 3/8 to improve monitoring technology from ½ to 4/5.

(5) The worker will provide $e=1$ rather than $e=0$ if $w+b-0.25>w+(1-0.5)b$, or when $b\geq 0.5$. Therefore, to induce $e=1$, the employer has to offer at least $b=0.5$. To make the contract acceptable to the worker, the wage should satisfy the participation constraint $w+0.5-0.25>0.85$, or $w\geq 0.6$. Therefore, to induce the agent to participate, the employer has to offer at least $w=0.6$. The employer will choose w and b to maximize her profits. The profits are equal to $1–w-b=1-0.6-0.5=-0.1$ if the worker provides $e=1$ and $-0.6-(1-0.5)0.5=-0.85$ if the worker provides $e=0$. Therefore, it is efficient for the employer not to employ the worker and obtain the outside option of 0. Any w and b that do not satisfy the worker's participation constraint will do.

(6) From the first-order condition $E[q'(e^*)]=c'(e^*)$, or $6=2e^*$, the efficient level of effort is $e^*=3$ in each period. To induce this level of effort in the second period, the wage must satisfy the no-shirking condition $w_2-3^2\geq 0.5w_2$, which implies that $w_2=18$. The expected utility in the second period is then $w_2-c(e_2)=18-3^2=9$. To induce the efficient level of effort in the first period, the wage must satisfy the no-shirking condition $w_1-3^2+9\geq 0.5w_1+0.5*9$, or $w_1=9$.

(7) From the first-order condition $E[q'(e_1^*)]=c'(e_1^*)$, or $6=e_1^*$, the efficient level of effort in the first period is $e_1^*=6$. Similarly, from the first-order condition $E[q'(e_2^*)]=c'(e_2^*)$, or $4=2e_2^*$, the efficient level of effort in the second period is $e_2^*=2$. To induce $e_2^*=2$ in the second period, the wage must satisfy the no-shirking condition $w_2-2^2\geq 0.5w_2$, which implies that $w_2=8$. The expected utility in the second period is then $w_2-c(e_2)=8-2^2=4$. To induce the efficient level of effort $e_1^*=6$ in the first period, the wage must satisfy the no-shirking condition $w_1-0.5*6^2+4\geq 0.5w_1+0.5*4$, which implies that $w_1=32$.

(8) As in Problem 6, the efficient level of effort is $e^*=3$ in both periods. In addition, the wage offer in the second period remains the same, $w_2=18$, as well as the expected utility in the second period, $w_2-c(e_2)=18-3^2=9$. What is different now is the no-shirking condition in the first period. Specifically, now we have $w_1-3^2+(0.9)*9\geq 0.5w_1+0.5*(0.9)*9$, which implies that $w_1=9.9$. Because the employee now discounts future gains, the employer needs to raise the current wage to compensate for this.

(9) The worker would have an incentive to stay with the firm as long as his wage is greater than his productivity and quit thereafter. The firm would therefore lose money on every worker. In addition, the reputation concerns may be important for the firm, but they may be difficult to establish for each individual worker.

(10) See the article.