Homework 8
Teams

(Q3 corrected)

Problems

(1)* Two accountants are about to form a partnership. The revenues of the partnership Q depend on the effort of each accountant according to $Q=e_1+e_2+u$, where u is a random variable with a mean of zero. The cost of effort is identical for both accountants: $c(e_i)=0.5e_i^2$. The accountants decided that they will split the revenues equally. What is the loss in the expected social surplus due to the free rider problem?

(2)** Consider a partnership of N agents, each with the cost of effort function $0.5e_i^2$. Each agent contributes $q_i=e_i$ to the total output $Q= e_1+e_2+ \dots +e_n+ u$, where u is a random variable with a mean of zero. Show that the loss in the social surplus due to the free rider problem increases in the size of partnership N.

(3)** Consider the partnership described in Problem 1. Suppose now that the first accountant receives β of the revenues while the second partner receives the remaining $1-\beta$ of the revenues. Show that the social surplus when the accountants use this sharing rule is smaller than the socially optimal surplus.

(4)* Consider the partnership described in Problem 1. Suppose now that the two accountants can impose a penalty on each other, given by $\gamma(G-e_i)^2$, where γ denotes guilt (or shame) and G is the group norm. Explain how would the partners set the group norm and how does the norm depend on γ.

(5)** Employee's output in Coca Cola is given by q=e+u, where e is effort and u is a random variable with a mean of zero. Each Coca Cola employee is paid individually according to w=q. Employees in Pepsi work in teams of 2. Each employee's output is q=e+n+u, where n is employee's ability. Pepsi employees get equal share of their total output $Q=q_1+q_2$. The cost of effort function is $0.5e^2$ for employees in both Coca Cola and Pepsi. All employees are risk neutral and don't exert peer pressure on each other. All employees in the economy are of two ability types: n=0 or n=1. What is the predicted observed difference in productivity per employee between Coca Cola and Pepsi?

Solutions

(1) The efficient level of effort for each accountant satisfies the first-order condition $E[q'(e_i^*)]=c'(e_i^*)$, or $1=e^*$. Therefore, the expected social surplus is $E[Q]-c(e_1)-c(e_2)=1+1-0.5(1^2)-0.5(1^2)=1$. If the partners share the revenues equally, the privately optimal level of effort for each partner satisfies the first-order condition $0.5E[q'(e_i^*)]=c'(e_i^*)$, or $0.5=e^*$. Therefore, the social surplus becomes $E[Q]-c(e_1)-c(e_2)=0.5+0.5-0.5(0.5^2)-0.5(0.5^2)=3/4$. Therefore, the expected loss in the social surplus is $1-3/4=1/4$.

(2) The efficient level of effort for each agent satisfies the first-order condition $E[q'(e_i^*)]=c'(e_i^*)$, or $1=e^*$. Therefore, the expected social surplus is $E[Q]-\sum_i c(e_i)= \sum_i (e_i-0.5e_i^2)=N(e-0.5e^2)=N[1-0.5(1^2)]=N/2$. If the agents share the revenues equally, the privately optimal level of effort for each agent satisfies the first-order condition $(1/N)E[q'(e_i^*)]=c'(e_i^*)$, or $1/N=e^*$. The social surplus is then $E[Q]-\sum_i c(e_i)= \sum_i (e_i-0.5e_i^2)=N(e-0.5e^2)=N[(1/N)-0.5((1/N)^2)]=1-1/2N$. Therefore, the expected loss in the social surplus is $N/2-1+1/2N=[(1+N^2)/2N]-1$. The derivative of this with respect to N is $(2N^2+1)/(2N)^2$, which is positive. Therefore, the expected loss in the social surplus due to the free rider problem increases with the size of the team N.

(3) From Problem 1, the socially optimal surplus is 1. Here, with this sharing rule, the privately optimal level of effort for partner 1 satisfies the first-order condition $\beta E[q'(e_1^*)]=c'(e_1^*)$, or $\beta=e_1^*$. For partner 2, the first-order condition is $(1-\beta)E[q'(e_2^*)]=c'(e_2^*)$, or $1-\beta=e_2^*$. Therefore, the social surplus becomes $E[Q]-c(e_1)-c(e_2)= \beta+1-\beta-0.5(\beta^2)-0.5(1-\beta)^2=1-0.5\beta^2-0.5(1-\beta)^2$. This expression achieves its maximum when $\beta=0.5$ since the first-order condition for β is $-\beta+(1-\beta)=0$. Evaluated at $\beta=0.5$, the social surplus becomes $1-0.5(0.5)^2-0.5(0.5)^2=1-0.5^2=0.75$, which is smaller than the socially optimal surplus of 1.

(4) Each accountant i now solves Max $0.5(e_i + e_j)-0.5e_i^2-\gamma(G-e_i)^2$. The first-order condition for e_i is $0.5-e_i+2\gamma(G- e_i)=0$. The accountants will choose the group norm G to induce the efficient level of e, $e^*=1$. Solving the first-order condition for G then yields $G=1+(1/4\gamma)$. Therefore, the norm is greater than the efficient level of effort 1. The norm also depends negatively on γ. Specifically, the more shame or guilt the partners feel about deviating from the norm, the smaller the norm can be to achieve the efficient level of effort.

(5) Employees in Coca Cola of any ability level will choose $e^*=1$. Their expected utility is then $1-0.5(1^2)=0.5$. An employee of ability n in Pepsi will choose $e=0.5$. The expected total output is then $Q=0.5+n+0.5+n=1+2n$. Therefore, if the worker is of ability $n=1$, his expected utility is $0.5(1+2)-0.5(0.5^2)=1.375$, which is greater than 0.5 that he can get in Coca Cola. On the other hand, if $n=0$, the employee's expected utility is $0.5(1+0)-0.5(0.5^2)=0.375$, which is less than 0.5 that they can get in Coca Cola. Therefore, employees with ability $n=0$ work for Coca Cola and employees with $n=1$ work for Pepsi. Productivity of Coca Cola employees is then 1 and the productivity of Pepsi employees is $0.5+1=1.5$.