Homework 7

Tournaments

Problems

(1)* Curling is one of the largest amateur sports in Canada with over one million active curlers. The most popular curling tournament is the Scotties Tournament of Hearts. The final game last year was between Team Alberta and Team Manitoba. Let q be the entertainment value of watching the game, which depends on the total number of points each team scores ($q=q_{AB}+q_{MB}$). The number of points each team scores depends on the team's effort e_i according to $q_i=e_i+u_i$, where i=AB, MB and u_i is a random variable with a mean of zero. The random variable $u_{AB}-u_{MB}$ is distributed uniformly on [-2,2]. The cost of effort to each team is $0.5e_i^2$. Both teams are risk neutral. What is the optimal prize spread between winning and losing that would induce both teams to choose efficient level of e?

(2)* Chicken raised for meat are called broilers[1]. Most broilers are produced by contract growers who are paid per pound of live broiler produced. The price per pound is determined by a tournament based on each grower's 'settlement' cost, which is the sum of feed and medical costs. Consider the following tournament between two growers. The settlement cost savings are given by $q_1=e_1+0.5u$ and $q_2=e_2-0.5u$, where u is a random variable with a mean of zero that is distributed uniformly on [-1, 1]. Suppose that the cost of effort is $0.5e_i^2$ for both growers i=1,2. The grower with the higher cost savings gets P per pound of live broiler while the grower with the lower cost savings gets p, where P>p. What is the optimal spread P-p? Who is more likely to win the tournament, grower 1 or grower 2?

(3)** A tournament is called fair and even if the contestants are identical and the rules don't favour one contestant over the other. For example, suppose that a man and a woman compete for a promotion. Assume that $q_M=e_M+0.5u$, $q_W=e_W-0.5u$, $c(e_i)=0.5e_i^2$, u is distributed uniformly on [-1,1], both contestants are risk-neutral, and both have the outside option of R=9.5. The tournament rule is that the man wins if $q_M>q_W$; otherwise, the woman wins. What is the optimal effort level for each contestant? What are the winning and losing prizes, W and w, required to make the tournament attractive for both contestants?

(4)** A tournament is called even but unfair when the contestants are identical but the rules favour one over the other. Consider the model in question 4, but now assume that the tournament rule is that the man wins if $q_M>q_w-k$, where k=0.5; otherwise, the woman wins. Therefore, this tournament favours the man over the woman. What is the optimal effort level for each contestant in this case? What are the winning and losing prizes, W and w, required to make the tournament attractive for both contestants?

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[1] For empirical analysis of tournaments in broiler production, see Charles Knoeber, "A Real Game of Chicken: Contracts, Tournaments, and the Production of Broilers", *Journal of Law, Economics, &Organization*, Vol. 5, No. 2. (Autumn, 1989), pp. 271-292.

(5)** Can you explain the following puzzle using insights from the tournament theory? "It is a common practice to announce a competition in order to select the design for a landmark building, such as the extension of Royal Ontario Museum. Such a competition, however, seem to be wasteful because it discards the effort by all architects who don't win the competition."

(6)* Tournament theory predicts that politicians will spend more effort on their campaigns in provincial as compared to municipal elections because of differences in gains from winning each type of elections. A study that randomized politicians into provincial and municipal elections strongly confirmed this result. However, a study that tracked politicians over time in provincial and municipal elections and that controlled for fixed differences among politicians indicated that there is no significant relation between politicians' effort and which type of election they participated in. Which of these two results are you likely to believe? Why?

(7)** Ronald Ehrenberg and Michael Bognanno[2] (EB) use data from the 1987 European Men's Professional Golf Association Tour to test implications of the tournament theory. Their results are presented in the following table:

Dependent variable = Final Score for the 1987 PGA European Tour

Variable	Description	Coefficient	t-value
TPRIZE	Total tournament prize money	-0.050	10.6
MAJ	Major tournament (=1 if yes)	-1.177	2.0
PAR	Par for the tournament course	2.411	6.0
YARDS	Course yardage	0.004	2.4
SAVE	Player's average score on all rounds during the 1987 tour	3.026	16.9

Notes: In golf, a lower final score reflects better performance. PAR and YARDS are controls for the difficulty of the course; SAVE is a proxy for player's ability.

(a) Are these results consistent with the tournament theory?
(b) Why do EB attempt to control for the player's ability (SAVE)? How would interpretation of the coefficient of TPRIZE change if they omitted SAVE?
(c) EB estimated their model by pooling data across players and tournaments. Explain how could you use these data in a fixed-effects model and how could this improve your inference.

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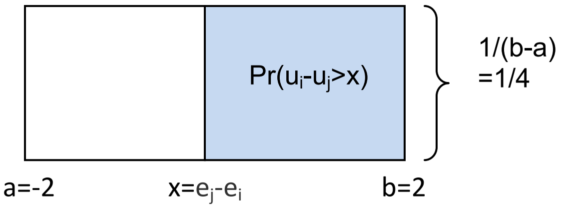
[2] Ronald Ehrenberg and Michael Bognanno, "The Incentive Effects of Tournaments Revisited: Evidence from the European PGA Tour", *Industrial and Labor Relations Review*, Vol. 43, No.3, (Feb, 1990), pp. 74S-88S.

Solutions

(1) We can approach this problem in three steps. First, the efficient level of effort for each team satisfies the first-order condition $E[q'(e^*)]=c'(e^*)$, which in this case implies $e^*=1$ for both teams since they have identical production and cost of effort functions. Second, the expected utility for team *i* is $E[U]=w+P(e_i,e_j)*(W-w)-\ 0.5e_i^2$. Therefore, the first-order condition for e_i is $\partial P(e_i,e_j)/\partial\ e_i*(W-w)-\ e_i=0$. This is the incentive compatibility constraint. To find $\partial P(e_i,e_j)/\partial\ e_i$, note that $P(e_i,e_j)=Prob(q_i>q_j)=Prob(u_i-u_j>e_j-e_i)$. Given that u_i-u_j has the uniform distribution defined on [-2,2], we have that $P(e_i,e_j)=(1/4)(2-e_j+e_i)$ – see the diagram below - and therefore $\partial P(e_i,e_j)/\partial\ e_i=1/4$. Therefore, the ICC becomes $(1/4)*(W-w)=e$. Third, to induce the efficient level of $e^*=1$, the ICC becomes $W-w=4$.

$Prob(u_i-u_j>x)=(b-x)/(b-a)$



(2) To find the optimal spread P-p, we can proceed as in Problem (1). First, the efficient level of effort for each grower satisfies the first-order condition $E[q'(e^*)]=c'(e^*)$, which in this case implies $e^*=1$ for both growers since they have identical production and cost of effort functions. Second, the expected utility for grower 1 is $E[U]=p+Pr(e_1,e_2)*(P-p)-\ 0.5e_1^2$. Therefore, the first-order condition for e_1 is $\partial Pr(e_1,e_2)/\partial e_1*(W-w)-e_1=0$. This is the incentive compatibility constraint. To find $\partial Pr(e_1,e_2)/\partial e_1$, note that $Pr(e_1,e_2)=Prob(q_1>q_2)=Prob(e_1+0.5u>e_2-0.5u)=Prob(u>\ e_2-e_1)$. Given that u has the uniform distribution defined on [-1,1], we have that $Pr(e_1,e_2)=(1/2)(1-e_2+e_1)$ and therefore $\partial Pr(e_1,e_2)/\partial e_1=1/2$. Therefore, the ICC becomes $(1/2)*(P-p)=e_1$. Third, to induce the efficient level of $e^*=1$, the ICC becomes $P-p=2$. Since the two growers are identical in terms of production and cost of effort, they will both supply the same efficient level of effort, $e^*=1$. Therefore, the probability that grower 1 wins is $Pr(e_1,e_2)=(1/2)(1-e_2+e_1)=(1/2)(1-1+1)=1/2$. Similarly, the probability that grower 2 wins is $1-Pr(e_1,e_2)=1-1/2=1/2$. Therefore, each grower is equally likely to win.

(3) Since the man and woman are identical in terms of production and cost of effort function, we can consider the man's problem as identical analysis applies for the woman. First, the efficient level of effort is 1. Second, the expected utility function is $E[U]=w+Pr(e_M,e_w)(W-w)-0.5e_M^2$. The first-order condition is $\partial P(e_M,e_W)/\partial\ e_M*(W-w)-\ e_M=0$. This is the incentive compatibility constraint. To find $\partial P(e_M,e_W)/\partial\ e_M$, note that $P(e_M,e_W)=Prob(q_M>q_W)=Prob(u>e_W-e_M)$. Given that u has the uniform distribution defined on [-1,1], we have that $P(e_M,e_W)=(1/2)(1-e_W+e_M)$ and therefore $\partial P(e_M,e_W)/\partial\ e_M=1/2$. Therefore, the ICC becomes $(1/2)*(W-w)=e$. Third, to induce the efficient level of $e^*=1$, the ICC becomes $W-w=2$. In addition, W and w must satisfy the participation constraint $E[U]=\ w+Pr(e_M,e_w)(W-w)-\ 0.5e_M^2=w+0.5(2)-0.5=w+0.5=R=9.5$, since $Pr(e_M,e_w)=0.5$ as both the man and woman choose the same effort level and have identical probability for winning. This implies that $w=9$. From ICC, we have that $W=w+2$, so $W=11$.

(4) As in problem (3), the efficient level of effort is 1 for both man and woman. Moreover, the expected utility for the man is $E[U_M]=w+Pr(e_M,e_w)(W-w)-0.5e_M^2$ and for the woman it is $E[U_W]=w+[1-Pr(e_M,e_w)](W-w)-0.5e_W^2$. The first-order conditions are therefore $\partial P(e_M,e_W)/\partial e_M*(W-w)-e_M=0$ for the man and $-\partial P(e_M,e_W)/\partial e_W*(W-w)-e_W=0$ for the woman. In addition, $Pr(e_M,e_w)=Pr(q_M>q_w-0.5)=Pr(e_M+0.5u>e_W+0.5u-0.5)=Pr(u>e_W-e_M-0.5)=(1/2)(1-e_W+e_M+0.5)$. Therefore, $\partial P(e_M,e_W)/\partial e_M=1/2$ and $-\partial P(e_M,e_W)/\partial e_W=1/2$. Therefore, the ICC is identical for both man and woman, $(1/2)(W-w)=e^*=1$, which implies that the optimal prize spread is $W-w=2$. Now, with $e_W=e_M=1$, we have that $Pr(e_M,e_w)=(1/2)(1-e_W+e_M+0.5)=0.75$ and $1-Pr(e_M,e_w)=0.25$. Therefore, the expected utility for the woman is $w+0.25(2)-0.5=w=R=9.5$. From ICC, we have that $W=w+2$, so $W=9.5+2=11.5$. We also have to verify that $w=9.5$ and $W=11.5$ satisfy the man's participation constraint. The man's expected utility with these prizes is $9.5+0.75(2)-0.5=9.5+1=10.5>R=9.5$, so the participation constraint is satisfied for the man as well.

(5) This is an example of all-or-nothing tournament in which the winner takes it all. This type of tournament may improve efficiency because it induces all agents to work hard to win the tournament. The waste of effort of other agents is apparent only – given the prize structure, each agent participates in the tournament voluntarily; i.e. they take the cost of effort and the probability of losing into consideration. An example of the production function with N contestants where the all-or-nothing tournament can work is $Q=\max\{q_1,q_2,\ldots,q_N\}$, so total output equals the output of the best agent only.

(6) Randomized experiments will identify the causal effect of interest (in this case, the impact of prizes on the effort level) when participants in the treatment and control group are sufficiently similar. Therefore, as long as randomization was done properly, this is likely to provide credible results. The fixed-effects strategy relies on two assumptions to identify the causal effect: 1. the only difference between the treatment and control group is a fixed factor that doesn't change over time, and 2. unobserved factors that change over time vary similarly for both the treatment and control group. We would need to examine to what extent these are reasonable assumptions by thinking about what factors may have changed over time that could have influenced both how hard politicians work on their campaigns and which election the politicians choose to participate in.

(7)

a. The tournament theory predicts that participants in the tournament will 'work' harder when the prize spread is higher. The results suggest that this may be the case: the final score is significantly smaller (t-value >2) both when the winning prize is higher and when it is a major tournament. However, the tournament theory claims that participants will work harder because of better incentives. From the results presented in the table, it is difficult to say whether they represent incentive or selection effects.
b. EB control for SAVE because it can influence both the outcome (the final score) and who participates in the tournament. Without controlling for SAVE, it would be more difficult to say whether the final score in tournaments with higher prizes is lower because players try harder or because more able players participate in the tournament.
c. Given that EB have information on both tournaments and players, they could include players' and tournaments' fixed effects which would account for any time invariant differences across players (e.g. ability) and tournaments (e.g. difficulty of the tournament).