Homework 6
Subjective Evaluation

Problems

(1)* Long-term success of the Republican Party in the U.S. depends in part on Sarah Palin's promotional effort according to q=e+u, where e is Sarah's effort and u is a random variable with a mean of zero. Sarah's compensation is a pay for performance contract w=a+by, where y is the percent of U.S. citizens who claim they would vote Republican if the election was held today. Sarah is risk-neutral and her cost of effort is given by $0.5e^2$. The Republican Party knows that y is proportional to q according to y=kq, where k has a mean of 1 and a variance of 3. On the other hand, Sarah knows and can manipulate the value of k. Find the optimal value of b. Show that this b induces an inefficiently low effort unless k=4.

(2)** Employee's contribution to the firm is given by q=e+u, where e is employee's effort and u is a random variable with a mean of zero. The cost of effort function is $e^2/6$. Both the employee and employer are risk neutral and have an outside option of zero. Compare the value of the expected social surplus when the employer uses q as a performance measure and when the employer uses y=kq as a performance measure, where k is a parameter that the employee can manipulate. The employee knows that the actual value of k is 1, while the employer knows only that k has a mean of 1 and a variance of V=1.

(3)* A cardiovascular surgeon can perform q surgeries per day according to q=4e+u, where e is physician's effort and u is a random variable with a mean of 0 and a variance of 2. The physician's disutility of effort is $c(e)=0.5e^2$ and his outside option is \$0. The physician is risk averse, with the coefficient of absolute risk aversion equal to 3. The Toronto General Hospital which employs the surgeon is risk-neutral. How much would the TGH be willing to pay to obtain a subjective performance measure that would completely eliminate uncertainty about u?

(4)* The quality of General Motors' cars depends on the assembly line worker's effort e according to q=10e. The cost of effort to the worker is e^2 and his outside option is R=10. The GM management can observe worker's effort but it cannot verify it to a third party. The payment contract for the worker is S if his effort level is at or above the efficient effort level and 0 otherwise. Show that the GM management has an incentive to misreport the actual worker's effort if it expects the company to go bankrupt after one period.

(5)* Consider the model described in question 4. Suppose that the U.S. government provides a large subsidy to GM, which saves it from bankruptcy. The GM now expects to be in business forever. The GM management realizes that if it misreports the actual effort of the worker, it will not be able to hire any workers in the future. If the GM management discounts the future profits at rate β=0.9, will it develop its reputation as a good firm or will it shut down after one period?

(6)* Consider the model of reputation concerns discussed in the class. Discuss how the principal's incentive to report truthfully depends on the agent's outside option R.

(7)* Sam can allocate his time between studying (e) and influencing his professors to get better grades (i). Sam's GPA is given by by $s=3q+i$, where q represents his knowledge, which is given by $q=e$. The cost of studying is $1.5e^2$ and the cost of influencing professors is i^2. What is Sam's expected GPA? How would his GPA and his knowledge change if the University implemented a strict measure that prohibited any influence activities?

(8)* Explain why fixed bonus pools are rarely used by companies.

Solutions

(1) The expected utility is $E[U]=E[w]-c(e)=a+bEy-0.5e^2=a=bke-0.5e^2$. The first-order condition for e is bk=e. This is the ICC. The PC is E[w]=R+c(e). The expected profits, using the ICC and PC, are $E[\Pi]=E[e]-E[0.5e^2]=E[bk]-E[0.5b^2k^2]=b-0.5\ b^2E[k^2]=b-0.5b^2(1+V)=b-2b^2$. The first-order condition for b is 1-4b=0. Therefore, b=0.25. From the ICC, this implies that e=bk=0.25k. The efficient level of effort solves E[q'(e*)]=c'(e*), or 1=e*. Therefore, unless k=4, we have that e≠e*.

(2) When the employer uses q as a performance measure, he can induce the efficient level of effort by using a piece rate contract w=a+q. This level of effort is given by the MB(e)=MC(e) condition, which in this example is 1=e*/3, so e*=3. The value of social surplus is then $E[q(e^*)]-c(e^*)=3-3^2/6=1.5$. When the employer uses y as a performance measure, the employee maximizes $a+bE[y]-c(e)=a+bke-e^2/6$. The ICC condition is then bk-e/3=0 or e=3bk. The employer maximizes his expected profit subject to ICC and PC. The expected profit function is $E[e]-c(E[e]) = 3bE[k]-E[(3bk)^2]/6 = 3b-1.5b^2E[k^2] = 3b-1.5b^2(1+V) = 3b-3b^2$. The optimal value of b is then 0.5. Given b=0.5, the employee will choose e=3bk=1.5. The social surplus when the employer uses y is then $1.5-1.5^2/6=1.125$.

(3) With no uncertainty about u, the hospital can implement the efficient level of effort given by MB(e*)=MC(e*). In this problem, this equals to e*=4. From the participation constraint, the salary level must satisfy $s^*=R+c(e^*)=0+0.5(4^2)=8$. Therefore, the hospital's profit in this case is E[q]-E[w]=4(4)-8=8.
When there is uncertainty, the hospital uses a piece-rate contract a+bq. To determine the value of this contract, we can proceed in three steps. First, the expected utility is $E[U]=E[w]-RP-c(e)=a+4be-0.5rb^2V-0.5e^2$. Therefore, the first-order condition for e is 4b-e=0. This is the ICC. Second, the PC is E[w]=R+c(e)+RP. Lastly, we can substitute for ICC and PC in the expected profit function $E[\Pi]=E[q]-E[w]=4e-0.5rb^2V-0.5e^2-R=16b-0.5rb^2V-8b^2$. The first-order condition for b is then 16-rbV-16b=0, from which it follows that b=16/(16+rV)=16/(16+3(2))=16/22≅0.7. From the participation constraint, we then have that $a+4be-0.5rb^2V-0.5e^2=0$, which yields $a=0.5(3)(0.7^2)(2)+8(0.7^2)-4(0.7)(4*0.7)$ ≅5.8. Therefore, the physician total expected compensation is E[w]=a+4be≅13.9. The expected profit is then E[q]-E[w]=4(4)(0.7)-13.9<0 and the hospital would not employ the physician and its profit would be 0. Therefore, the maximum the hospital would be willing to pay to remove all uncertainty is 8.

(4) The efficient level of effort is given by the MB(e*)=MC(e*) condition, which in this case yields e*=5. The salary level is then determined from the participation constraint as $S=R+C(e)=10+5^2=35$. The GM profit for one period if it reports e<5 is 10(5)-0=50 and its profit if it reports e≥5 is 10(5)-35=15. Therefore, if GM expects to go bankrupt after one period, it will likely under-report the worker's true effort.

(5) If the GM management misreports the worker's actual effort, it will earn 50 this period and then it will earn zero forever since no worker will want to work for it. If the GM management chooses to report truthfully, its profit will be 15/(1-0.9)=150. Therefore, it is in the GM management's best interest to develop reputation as a good firm.

(6) In the model of reputation concerns, the gain from misreporting is given by E[q]. The gain from reporting truthfully is given by $(E[q]-c(e)-R)/(1-\beta)$. Therefore, all else equal, the incentive to report truthfully is smaller when R is higher.

(7) Sam will choose e and i to maximize his utility, given by $s-c(e)-c(i)=3e+i-1.5e^2-i^2$. The first-order conditions for e and i are 3-3e=0 and 1-2i=0, respectively, which yields e=1 and i=0.5. Therefore, his knowledge is q=1 and his GPA is s=1(3)+0.5=3.5. In the absence of any influence activity, his GPA would be s=3, but his knowledge q would be 1 as before.

(8) The answer is based on Bol (2005). The Fixed bonus pools are rarely used by firms for at least three reasons: they discourage co-operation among employees; they encourage sabotage activities of employees; and they are not effective in solving other problems related to rating bias, such as favouritism and rent-seeking.