Homework 5
Multiple Signals and Multiple Tasks

Problems

A. Multiple Signals

(1)* Family Guy is an animated television sitcom created by Seth MacFarlane that airs on Fox. The Fox management is considering changing the compensation structure for MacFarlane from $w=a+bq$ to $w=a+bq+\theta s$, where q is the number of Fox viewers, s is the number of CNN viewers, and b=0.2. The expected number of Fox viewers is $E[q]=e$ and its variance is $Var[q]=V=1$, while the expected number of CNN viewers is $E[s]=0$ and its variance is $Var[s]=W=0.5$. In addition, the covariance between q and s is given by $cov(q,s)=\rho=-0.5$. MacFarlane is risk averse with the coefficient of absolute risk aversion equal to r=3. Find the risk premium for MacFarlane if the Fox management optimally uses signal s. Compare this risk premium to the risk premium when the Fox management does not use signal s.

(2)* University professors are currently compensated according to a pay for performance contract $w=a+bq$, where q is the number of research articles that the professor publishes in economic journals. The Board of Directors is considering introducing an additional performance measure in the professors' contract, such as students' evaluation scores. Explain how the Board should use this additional measure if the professors are risk neutral.

(3)* Suppose that the proposed contract is $w=a+b(q+\theta s)$, where q is agent's performance and s is another signal of agent's performance. Suppose further that $E[q]=E[s]=0$, $Var[q]=V$, $Var[s]=1$, and $Cov[q,s]= \rho$. What is the value of θ that minimizes the variance of agent's pay?

(4) ** Wolgang Puck, a celebrity chef from California, wishes to hire a manager for his new restaurant. Mr. Puck proposes to pay $w=a+bq+\theta s$, where q is the number of customers in his restaurant and s is the number of customers in a restaurant of similar quality. Specifically, Mr. Puck knows that $E[q]=e$, $Var[q]=V=2$, $E[s]=0$, $Var(s)=1$, and $cov(q,s)=\rho=0.9$. In addition, Mr. Puck knows that the manager's cost of effort is $0.5e^2$ and his coefficient of risk aversion is r=2. Show that the expected number of customers in Mr. Puck's restaurant is greater if the contract is based on both q and s rather than on q alone.

(5) ** Suppose that the agent's output is $q=e+u$, where e is effort and u is a random variable. Suppose further that $u=\beta s+v$, where s is an additional signal and $E[v]=0$. Show that if the principal pays the agent based on the expected effort given signal s, the optimal contract has the form $q=a+bq-b\beta s$.

B. Multiple Tasks

(6)* The hockey player's contribution to the team is given by $q=e_1+e_2$, where e_1 represents effort expended on scoring goals and e_2 represents effort expended on assisting others to score goals. The team manager can't observe directly either e_1 or e_2. However, he can observe the number of goals scored (y_1) and the number of assists (y_2). The number of goals scored is given by $y_1=e_1+u_1$, where u_1 is a random variable with a mean of zero and a variance of 1. Similarly, the number of assists is given by $y_2=e_2+u_2$, where u_2 is a random variable with a mean of zero and a variance of 1. Assume that u_1 and u_2 are not correlated. The player is risk averse with a coefficient of absolute risk aversion of 2 and cost of effort function $c(e_1,e_2)=0.5(e_1+e_2)^2$. The contract specifies that the player's total compensation equals $w=a+b_1y_1+b_2y_2$. What is the optimal compensation weight on goals scored (b_1) and assists (b_2)?

(7)* Your learning in this course is given by $q=e_1+ke_2$, where e_1 represents effort expended on attending classes, e_2 represents effort expended on doing homework problems, and $k>0$ is a constant that reflects the relative contribution on e_2 to your learning. Your professor cannot observe either e_1 or e_2. However, he can observe two signals, y_1 and y_2¸that are related to your effort on each task. Specifically, $y_1=e_1+u_1$, with $E[u_1]=0$ and $Var[u_1]=V_1$, and $y_2=e_2+u_2$, with $E[u_2]=0$ and $Var[u_2]=V_2$. You are risk neutral and your cost of effort is given by $c(e_1,e_2)=0.5(e_1^2+e_2^2)$. If your grade in this course is given by $w=a+b_1y_1+b_2y_2$, show that the professor will set $b_1>b_2$ only if $k<1$.

(8)* The Ministry of Health is interested in both quantity and quality aspects of physician practice. Specifically, the outcome is $q= e_1+e_2$, where e_1 represents effort expended on seeing more patients and e_2 represents effort expended on provide better quality service to each patient. The Ministry cannot observe either e_1 or e_2. However, it can observe two signals, y_1 and y_2¸that are related to the physician effort on each task. Specifically, $y_1=e_1+u_1$, with $E[u_1]=0$ and $Var[u_1]=V_1$, and $y_2=e_2+u_2$, with $E[u_2]=0$ and $Var[u_2]=V_2$. The physician is risk neutral and his cost of effort is given by $c(e_1,e_2)=0.5(e_1^2+ke_2^2)$. The contract specifies the physician pay as $w=a+b_1y_1+b_2y_2$. Find the optimal weights b_1 and b_2 and the effort levels that these weights induce.

(9)* Auto insurance agents often are expected to provide on-going customer service after a policy is sold (answering questions about the policy, providing assistance in filing claims, and so on). Explain why some auto insurance companies separate the tasks of selling and customer service. Which of the two jobs should be tied more to measurable performance?

(10*) Jane Hannaway (1992) proposes to separate the jobs of teaching basic and higher skills. How does she respond to the following objections to her argument?

- It is inefficient to separate jobs because they are complementary tasks;
- Separating the job may introduce co-ordination problem between teachers.
- Teachers may not be motivated by economic incentives.

Solutions

A. Multiple Signals

(1) The risk premium is 0.5rVar[w], where r is the coefficient of absolute risk aversion. Therefore, this risk premium will be optimal by choosing θ to minimize Var[w]. We have that Var[w]=Var[a+bq+θs]=$b^2V+\theta^2W+2b\theta\rho$. The first-order condition for θ is $2\theta W+2b\rho=0$, which yields $\theta=-b\rho/W$. Therefore, the optimal risk premium is equal to $0.5r(b^2V+\theta^2W+2b\theta\rho)=0.5r(b^2V-b^2\rho^2/W)$. Substituting for values given in the question, this is equal to $0.5(3)(0.2^2 1-0.2^2(-0.5^2)/0.5)=0.03$. Without using signal s, the risk premium is $0.5rb^2V=0.5(3)(0.2^2)=0.06$.

(2) The potential benefit of using the additional signal is to reduce risk inherent in pay and therefore also improve incentives by increasing the value of b. However, when the agent (professors here) are risk neutral, they are not concerned with risk and the principal can set b to its optimal value of 1. There is no benefit of using any additional signals.

(3) The variance in this problem is Var[w]=Var[a+b(q+θs)]=$b^2(V+\theta^2+2\theta\rho)$. The first-order condition for θ is then $b^2(2\theta+2\rho)=0$, which yields $\theta=-\rho$.

(4) Consider first the contract that does not include s, i.e. w=a+bq. We have that E[U]=E[w]-0.5rVar[w]-c(e)=$a+be-0.5rb^2V-0.5e^2$. Therefore, ICC yields b=e. In addition, PC yields that E[W]=R+c(e)+0.5rVar[w]. Substituting these two constraints in the expected profit, we have E[Π]=E[q]-E[w]=$b-0.5rb^2V-0.5b^2-R$. The first-order condition for b is then 1-rbV-b=0, which gives b=1/(1+rV). Substituting for r and V from the problem, we then have that b=1/(1+2(2))=1/5=0.2. This is also the expected number of customers since from ICC, b=e. Consider now the contract that includes s, i.e. w=a+bq+θs. The expected utility function now becomes E[U]=E[w]-0.5rVar[w]-c(e)=$a+be-0.5r(b^2V+\theta^2+2b\theta\rho)-0.5e^2$. Therefore, from the first-order condition for b, ICC is still b=e. Also, we have from PC that E[W]=R+c(e)+0.5rVar[w]. Substituting these two constraints in the expected profit, we have E[Π]=E[q]-E[w]=$b-0.5r(b^2V+\theta^2+2b\theta\rho)-0.5b^2-R$. We now need the first-order conditions for both b and θ. For b, the FOC is $1-rbV-r\theta\rho-b=0$ and for θ we have $r\theta+rb\rho=0$. Therefore, $\theta=-b\rho$. Substituting in the FOC for b we then have that $1-rbV-r(-b\rho)\rho-b=0$, which yields $b=1/(1+rV-r\rho^2)$. Substituting for values of r, V, and ρ from the problem, we have that $b=1/(1+2(2)-2(0.9^2))\cong 0.3$. This is also the expected number of customers, since from ICC we have that b=e. Notice that this is higher than when s is not used. Lastly, we can also find θ from $\theta=-b\rho=-0.3(0.9)=-0.27$. Therefore, the optimal contract is w=a+0.3q-0.27s.

(5) The problem is that the principal observes only q and cannot separate the effect of e and u on q. However, the principal can learn something from observing s. In particular, E[u|s]=βs. Therefore, the principal can conclude after observing s that E[e|s]=q-

$E[u|s]=q-\beta s$. If the agent is paid based on the expected output, then the pay is $w=a+bE[e|s]=a+b(q-\beta s)=a+bq-b\beta s$. Now, in the class we had that the optimal weight on the signal is $\theta=-b\rho$. This implies that β in this example is equal to covariance between u and s, which is exactly the interpretation of the regression coefficient in the model $u==\beta s+v$ when the variance of s is 1.

B. Multiple Tasks

(6) The player's expected utility is $E[U]=E[w]-RP-c(e)=a+b_1e_1+b_2e_2-0.5r(b_1^2V_1+b_2^2V_2)-0.5(e_1+e_2)^2$. To find the ICC, note that the first-order conditions for e_1 and e_2 are $b_1=e_1+e_2$ and $b_2=e_1+e_2$, respectively. This implies that $b_1=b_2$. Let this common value be denoted by b. Also, let e_1+e_2 be denoted by e. Therefore, we can write the ICC as b=e. Also, the participation constraint is E[w]-RP-c(e)=R. Using the ICC and PC, we can finally write the expected profit function as $E[\Pi]=E[q]-E[w]=e-0.5rb^2(V_1+V_2)-0.5e^2-R= b-0.5rb^2(V_1+V_2)-0.5b^2-R$. The team manager maximizes this expected profit by choosing the value of b. The first-order condition is $1-rb(V_1+V_2)-b=0$, from which it follows that $b=[1+r(V_1+V_2)]^{-1}$. Substituting for the values of r=2 and $V_1=1=V_2$ given in the problem, this implies that $b=1/5=0.2=b_1=b_2$.

(7) Your expected utility is $E[U]=E[w]-c(e)= a+b_1e_1+b_2e_2-0.5(e_1^2+e_2^2)$. (Note that there is no risk premium since you are risk neutral.) You will choose your effort level to maximize your expected utility, so the first-order conditions for e_1 and e_2 are $b_1=e_1$ and $b_2=e_2$, respectively. This is the ICC. The PC is simply that $E[w]=R+c(e_1,e_2)$. Substituting for the ICC and PC in the expected profit function, we have $E[\Pi]=E[q]-E[w]= e_1+ke_2-0.5(e_1^2+e_2^2)-R = b_1+kb_2-0.5(b_1^2+b_2^2)-R$. The first-order conditions for b_1 and b_2 are $1-b_1=0$ and $k-b_2=0$, respectively. Therefore, $b_1=1$ and $b_2=k$ and $b_1>b_2$ only if $k<1$.

(8) The physician's expected utility is $E[U]=E[w]-c(e)= a+b_1e_1+b_2e_2-0.5(e_1^2+ke_2^2)$. (Note that there is no risk premium since the physician is risk neutral.) The physician will choose his effort levels to maximize his expected utility, so the first-order conditions for e_1 and e_2 are $b_1=e_1$ and $b_2=ke_2$, respectively. This is the ICC. The PC is simply that $E[w]=R+c(e_1,e_2)$. Substituting for the ICC and PC in the Ministry's expected profit function, we have $E[\Pi]=E[q]-E[w]= e_1+e_2-0.5(e_1^2+ke_2^2)-R = b_1+b_2/k-0.5(b_1^2+k(b_2/k)^2)-R$. The first-order conditions for b_1 and b_2 are $1-b_1=0$ and $1/k-b_2/k=0$, respectively. Therefore, $b_1=1$ and $b_2=1$ and from the ICC we have $e_1=1$ and $e_2=1/k$.

(9) Sales and customer service are both valuable to the company, but the quality of service is much harder to measure. If both tasks are valued, and service is hard to measure, then it is optimal not to provide any incentives to either task (i.e. salary). This is inefficient since the agent may not try hard on either task. The firm can do better by separating the tasks and paying salesmen based on the value of sales and paying customer service reps a fixed salary.

(10) See Hannaway (1992).