Homework 1

Salary Model

Problems

(1)* Ana needs to hire a contractor to renovate her house. For each hour of contractor's work *e*, the value of Ana's house increases by 10*e*. The contractor's disutility of work is $0.5e^2$. Ana's outside option is 0 and the contractor's outside option is 25. Is it efficient that Ana hires the contractor? If so, what contract should Ana offer to the contractor?

(2)* Bill wishes to hire a stockbroker to manage his portfolio. For each unit of stockbroker's effort *e*, the value of Bill's portfolio increases by 10ln(*e*). The stockbroker's disutility of effort is 5*e*. Bill's outside option is 0 and the stockbroker's outside option is 20. Is it efficient that Bill hires the stockbroker? If so, what contract should the stockbroker offer to Bill?

(3)* Claudette wants to hire a financial planner to prepare her retirement plan. For each unit of planner's effort *e*, the value of Claudette's retirement plan increases by 5*e*. The planner's disutility of effort is $5e^2$. Claudette's outside option is 0.25 and the planner's outside option is 0.5. Which of the following two contracts, if any, is efficient: $[e_1,s_1]$=[0.5,\$2] or $[e_2,s_2]$=[1,\$2.25]?

(4)** A physician can treat 20 patients for each hour of work *e.* The value of each treated patient to the Ministry of Health is 30. The physician's disutility of work is given by 500*e.* What number of hours should the physician work if the medical regulations stipulate that the physician can work no more than 8 hours per day? Assume that the outside options are such that it is efficient for the Ministry of Health to employ the physician.

(5)** A nurse can treat one patient for each hour of work *e.* The value of each treated patient to the hospital is 1. The nurse's disutility of work is $0.5e^2$ if she works one hour or less and e^2 if she works more than one hour. What is the efficient level of *e*? Assume that the outside options are such that it is efficient for the hospital to employ the nurse.

(6)** A worker produces output according to pq(*e*)=10*e*, where *e* is effort. The worker's cost of effort is $0.5e^2$. If *e* can take only four values: {0, 1, 2, 3), what is the efficient level of *e*? Assume that the outside options are such that it is efficient that the firm employes the worker.

(7)** Hospital for Sick Kids has to decide whether to hire a part-time or a full-time pharmacist.

a. Interpret this problem in terms of the principal-agent relationship.
b. Describe the principle that the hospital should use to guide its decision.
c. What compensation should the hospital offer to the new pharmacist?

(8)** You wish to renovate your bathroom. You have two options: a complete make-over (paint the walls and change the tiles) or a partial make-over (paint the walls only). You need to hire a home renovator to do the job.

a. Interpret this problem in terms of the principal-agent relationship.
b. Explain the principle that you would use to make your decision.
c. What data would you need to collect to make your decision?

(9)*** Workers usually have different attitudes toward work. How would you modify the salary model to account for this fact? What implications would this have for the design of optimal salary contracts?

Solutions

(1) In this case, Ana is the principal and the contractor is the agent. To decide whether Ana should hire the contractor, we proceed in two steps. First, we find the contract that maximizes principal's payoff and is also acceptable to the agent. Second, we verify that the principal is as no worse by hiring the agent than by not hiring him. Step 1: The efficient contract [e, s] maximizes Ana's payoff $\prod$=pq-s=10e-s. To be acceptable to the contractor, the contract must satisfy the contractor's participation constraint U=R, where U=s-c(e)=s-0.5e^2 and R=25. Therefore, s that satisfies the contractor's participation constraint is s=R+c(e)=25+0.5e^2. Substituting for s in Ana's payoff yields $\prod$=10e-s=10e-25-0.5e^2. This is now Ana's payoff that satisfies the contractor's participation constraint. Notice that this payoff depends only on the choice of e. The first-order condition for e is 10-e*=0, which implies that e*=10. Substituting for e*=10 in the contractor's participation constraint gives s*=25+0.5(10^2)=75. Therefore, the contract that maximizes Ana's payoff and is also acceptable to the contractor is [e*,s*]=[10,75]. Step 2: We'll verify that $\prod$(e*,s*) is at least as great as $\prod_R$. We have that $\prod$(e*,s*)=10e*-s*=10(10)-75=25. This is greater than Ana's outside option of 0. Therefore, it is efficient that Ana hires the contractor.

(2) In this case, Bill is the principal and the stockbroker is the agent. We can proceed as in the previous problem, except that now we wish to find a contract that maximizes the agent's payoff but is still acceptable to the principal. Step 1: The efficient contract [e, s] maximizes the stockbroker's payoff U=s-c(e)=s-5e. To be acceptable to Bill, the contract must satisfy Bill's participation constraint $\prod$=$\prod_R$, where $\prod$=pq-s=10ln(e)-s and $\prod_R$=0. Therefore, s that satisfies Bill's participation constraint is s=pq(e)-$\prod_R$=10ln(e). Substituting for s in the stockbroker's payoff yields U=s-c(e)= 10ln(e)-5e. This is now the stockbroker's payoff that satisfies Bill's participation constraint. Notice that this payoff depends only on the choice of e. The first-order condition for e is 10/e*-5=0, which implies that e*=2. Substituting for e*=2 in Bill's participation constraint gives w*=10ln(2)≅6.9. Therefore, the contract that maximizes the stockbroker's payoff but is still acceptable to Bill is [e*,s*]=[2,6.9]. Step 2: We'll verify that U(e*,s*) is at least as great as R=20. We have that U(e*,s*)=s*-5e*=6.9-5(10)=-3.1. Since this is smaller than the stockbroker's outside option of 20. Therefore, it is not efficient that Bill hires the stockbroker.

(3) In this case, Claudette is the principal and the financial planner is the agent. The problem does not specify who designs the contract, but we know that the choice of the efficient level of effort does not depend on who designs the contract. To find the efficient level of effort, we have to find e that maximizes the value of the relationship, pq(e)-c(e)=5e-5e^2. The first-order condition is 5-10e*=0 and therefore e*=0.5. To determine the salary acceptable to both Claudette and the planner, we use the participation constraint for each party. For Claudette, the constraint is that $\prod^* \geq \prod_R$, or 5e*-s*≥0.25, or s*≤5(0.5)-0.25=2.25. For the financial planner, the constraint is that

$U^* \geq R$, or $s^*-c(e^*)=s^*-5e^{*2} \geq 0.5$, or $s^* \geq 0.5+5(0.5^2)=1.75$. Therefore, the efficient contract has $e^*=0.5$ and $1.75 \leq s^* \leq 2.25$. Therefore, the contract $[e_1,s_1]=[0.5,\$2]$ is efficient, while the contract $[e_2,s_2]=[1,\$2.25]$ is not efficient.

(4) In this case, the physician is the agent and the Ministry of Health is the principal. To find the efficient number of hours of work, we have to compare the marginal benefit and the marginal cost of hours of work. The total benefit is $pq(e)= 30\times20e$, so the marginal benefit is 600. The total cost is 500e, so the marginal cost is 500. Therefore, the marginal benefit always exceeds the marginal cost, for any value of e. Therefore, the physician should work the maximum number of hours permitted by the medical regulations, 8. Note that in this case we cannot use the first-order condition $pq'(e^*)-c'(e^*)=0$ to find the efficient level of effort because both the total benefit and the total cost functions are linear ($q''=0=c''$ for all e).

(5) In this case, the hospital is the principal and the nurse is the agent. To find the efficient number of hours of work, we have to find e that maximizes the value of the relationship. This value is $e-0.5e^2$ for $e \leq 1$ and $e-e^2$ for $e>1$. In other words, this value is defined over two intervals of e. To find which e maximizes this value, we have first to find the optimal e for each interval. For $e \leq 1$, the first-order condition gives $1-e^*=0$, or $e^*=1$. For $e>1$, the first-order condition gives $1-2e^*=0$, or $e^*=0.5$. In the second step, we have to compare the value of relationship over each interval evaluated at the optimal e. For $e \leq 1$, this value is $1-0.5(1^2)=0.5$. For $e>1$, this value is $0.5-(0.5^2)=0.25$. Therefore, the efficient level of e is 1.

(6) The value of relationship $V=pq(e)-c(e)=10e-0.5e^2$. To find efficient e, we have to evaluate the relationship value at each distinct value of e. We have: $V(0)=0$; $V(1)=10-0.5(1^2)=9.5$, $V(2)=20-0.5(2^2)=18$, and $V(3)=30-0.5(3^2)=25.5$. Therefore, the efficient value of e is 3. Note that this choice is restricted because e can take only these four discrete values. If e was a continuous variable, the first-order condition for e is $10-e=0$, and the efficient level of e would therefore be 10.

(7) (**a**) In this case, the hospital is the principal and the pharmacist is the agent. Let pq represent the value that the hospital gets from employing the pharmacist. For example, this could be the number of additional patients the hospital can treat q, multiplied by the revenue per patient p. Effort takes two values, 0.5 and 1, representing the part-time pharmacist and the full-time pharmacist, respectively. It seems reasonable to assume that $pq(1)>pq(0.5)$. It also make sense to assume that $c(1)>c(0.5)$, where c() represents the cost of working for the pharmacist. From the hospital's perspective, c() represents the amount that the hospital has to pay above the pharmacist's outside option R to attract the pharmacist. (**b**) The hospital should hire a full-time pharmacist if and only if $pq(1)-c(1)>pq(0.5)-c(0.5)$, or when $pq(1)-pq(0.5)>c(1)-c(0.5)$. This last expression means that the hospital should hire the full-time pharmacist only if the value of the full-time pharmacist relative to the part-time pharmacist exceeds what the hospital has to pay to attract the full-time pharmacist. When the opposite condition holds, the hospital should

employ the part-time pharmacist. (**c**) The hospital should offer s=R+c(1) and employ a full-time pharmacist if pq(1)-pq(0.5)>c(1)-c(0.5). Otherwise, the hospital should employ a part-time pharmacist and offer s=R+c(0.5).

(8) (**a**) In this case, you are the principal and the renovator is the agent. Let pq represent how much you value each option. The two options represent two different values for e, say A and B, that denote the complete make-over and the partial make-over, respectively. It seems reasonable to assume that pq(A)>pq(B) and that c(A)>c(B), where c() represents the cost of completing each job for the renovator. From your perspective, c() represents additional price you have to pay above the renovator's outside option R to make the offer attractive to him. (**b**) Similar to the previous problem, you would choose option A over option B if and only if pq(A)-c(A)>pq(B)-c(B), or when pq(A)-pq(B)>c(A)-c(B), or when how much you value option A over option B exceeds the additional cost you have to pay to do option A rather than option B. (**c**) To make your decision, you would need to collect data on the difference in cost between options A and B, c(A)-c(B). You would then compare whether this difference is smaller or larger than the difference in how much you prefer option A over option B.

(9) One approach to incorporate this fact in the model would be to specify the cost of effort function as c(e)=kv(e), where v(e) is the disutility of effort common to all workers and k is a parameter that varies among workers. With this specification, workers with higher k have a higher cost of effort. The model we studied in the class can be considered a special case with k=1 for all workers. This heterogeneity among workers can affect the optimal contract in two ways. First, the optimal level of effort now satisfies the first-order condition pq'(e*)=kv'(e*). Therefore, the contract would specify a lower level of effort for workers with a higher cost of effort. Second, the compensation may also vary among workers since s=R+kv(e). However, it is not clear whether workers with a lower or higher cost of effort would be compensated more. The reason is that s depends on both k and v(e) and while k is higher for workers with a higher cost of effort, v(e) is lower compared to workers with a lower cost of effort. To see this, consider two workers, one with k_0 and the other with k_1, with $k_0<k_1$ (in this example, salaries are identical, but this is clearly not always the case):