Garibaldi, 132 - 138

8 Relative Compensation and Efficiency Wage

8.1 Introduction

Piece rates and bonus schemes provide incentives that operate independently of the relationship among co-workers. Specifically, workers need not be working with anyone else to be motivated by a piece rate scheme. Piece rate compensation is based on an individual's *absolute performance* rather than his performance relative to some standard or some other individual. Yet, in reality, most individual motivation is produced not by absolute reward but by a compensation that is based on *relative comparison*. Managerial employees who move up the corporate ladder do so by being better than their peers, not necessarily by being good.

Comparisons are key in determining promotions in private firms. Since promotions carry with them higher salaries, higher status, and perhaps more interesting assignments, workers seek to get promotions. In this process, they exert effort in an attempt to outperform their neighbours. Thus relative compensation can provide an incentive that is as effective as a piece rate or output-based compensation scheme based on individual performance.

There are good reasons why firms may prefer to use relative compensation schemes. The first is that it may be easier to observe relative position than it is to observe absolute position. Second, relative compensation differences out common noise that risk-averse people may not like.

The most extreme form of relative compensation is a pure tournament, a setting in which two workers compete for a promotion, an outcome that can be described in terms of higher salaries. Workers are promoted if they do better than their co-workers. In such a setting, the effort exercised by the workers is proportional to the spread in prize, or to the increase in salary associated with the promotion.

Pure tournaments are just one possible relative compensation scheme. Indeed, the chapter introduces also the concept of a Linear Performance Evaluation (LPE), a situation in which workers receive a salary structure that is the sum of two components: a fixed part plus a bonus (penalty) term, with the bonus term linked to the average performance of the workers. There are differences between LPE and pure tournament, particularly important when players are heterogeneous. In LPE winning by a lot is important, since the

larger the victory the larger the increase in salary. In pure tournaments, conversely, all that is important is winning. The implications of the two relative compensation schemes are analysed in the context of a case study of broiler production.

The chapter introduces also the concept of efficiency wage, a compensation scheme whereby firms use salaries to increase workers' effort in the context of imperfect information. In a dynamic context, the efficiency wage mechanism ensures that workers' wages increase with tenure, even when productivity is constant. Upward-sloping wage profiles appear particular relevant in real life labour markets. Further, they form the basis of implicit contract between workers and firms, whereby workers continuously exercise effort in view of future wage increases.

The chapter proceeds as follows. Section 8.2 presents the basis tournament model. Section 8.3 introduces the relative compensation scheme, while sections 8.4 and 8.5 are devoted to the case study of broiler production. Section 8.6 deals with efficiency wages and section 8.7 with the upward-sloping wage profiles.

8.2 Tournament: A Formal Story

Relative compensation theory, or tournament theory as it has come to be called, is the theory used to determine the size of a rise associated with a particular promotion. It has four essential features.

1. Prizes are fixed in advance and are independent of absolute performance. The key is just winning the contest, not playing well, and typically the prize awarded is independent of the difference between the two players. In the context of the firm, this means that there are wage slots that are fixed in advance.
2. A player receives the winner's or loser's prize not by being good or bad but by being better than, or worse than, the other player. Again relative performance rather than absolute performance is key.
3. The effort with which the worker pursues the promotion depends on the size of the salary increase that comes with the promotion.
4. It is important to recall that the spread must be large to induce effort, but the average prize money must be sufficiently high to attract workers to come to the firm in the first place. Otherwise, workers will opt to enter some other activity, since participation is not mandatory.

Consider a firm that has only two workers and sets up two jobs: boss and operator. Workers compete against one another with the winner being designated boss and the loser being designated operator. The winner receives wage

W_1, and the loser receives wage W_2. No wages are paid until after the contest is completed. The probability of winning the contest depends on the amount of effort that each individual exerts. Let the two individuals be denoted by j and k.

Output of worker j is denoted by q_j and depends on effort e_j and on luck z so that

$$q_j = e_j - \frac{1}{2}z,$$

where z is a random variable with 0 mean. Similarly, output of worker k is

$$q_k = e_k + \frac{1}{2}z$$

where z can simply be interpreted as relative luck with 0 mean. This implies that expected output for worker j is $E(q_j) = e_j$ while expected output of worker k is equal to $E(q_k) = e_k$. Effort is chosen by the worker and it is not observed by the firm, which observes only output.

We assume that z is uniformly distributed with zero mean over the interval $[-b, b]$. Recall that if a variable is uniformly distributed over the interval $[-b, b]$ the following features hold:

$$E(z) = 0$$

$$Var(z) = \frac{b^2}{3}$$

$$G(x) = P(z \leq x) = \frac{x+b}{2b}$$

$$g(x) = \frac{1}{2b}$$

where G is the cumulative distribution function and g is its density function.

Let's define with P the probability of winning the context. For worker j, the probability of winning the context depends on whether his output j is larger than output k, so that

$$\begin{aligned} P(q_j > q_k) &= \Pr ob\left(e_j - \frac{1}{2}z > e_k + \frac{1}{2}z\right) \\ &= \Pr ob(z < e_j - e_k) \\ &= G(e_j - e_k) \\ &= \frac{e_j - e_k + b}{2b} \end{aligned}$$

We are now in a position to solve the problem. We do this in two steps. First, we model worker behaviour. Second, we solve for the firm maximization

problem, taking worker behaviour into account. The second problem is thus choosing an optimal compensation scheme to maximize profits.

8.2.1 WORKER PROBLEM

The worker utility is the expected wage minus the cost of effort. We assume that the cost of effort is quadratic in effort, while the worker is risk neutral. In other words, if P is the probability of winning the contest the worker's pay-off is

$$U = W_1 P + (1-P)W_2 - \frac{\delta e_j^2}{2},$$

where δ is the slope of the marginal disutility of effort. The worker chooses e to maximize expected output so that his problem reads

$$Max_{e_j} \quad W_1 P(q_j > q_k) + [1 - P(q_j > q_k)]W_2 - \frac{\delta e_j^2}{2}$$

Since we know that $P(q_j > q_k) = G(e_j - e_k)$ the problem becomes

$$Max_{e_j} \quad W_1 G(e_j - e_k) + (1 - G(e_j - e_k))W_2 - \frac{\delta e_j^2}{2}$$

The FOC reads

$$\begin{aligned} (W_1 - W_2)\frac{\partial G}{\partial e_j} &= \delta e_j \\ (W_1 - W_2)g(e_j - e_k) &= \delta e_j \\ \frac{W_1 - W_2}{2b} &= \delta e_j \end{aligned}$$

where we used the fact that $\frac{\partial G(e_j - e_k)}{\partial e_j} = g(e_j - e_k)$ and that $g(e_j - e_k) = \frac{1}{2b}$ with a uniform distribution.

Note that there is a symmetrical problem for worker k which yields, after a simple and similar simplification,

$$(W_1 - W_2)g(e_j - e_k) = \delta e_k.$$

Since the workers are *ex ante* identical and have identical first-order conditions, there should be a *symmetrical equilibrium in which both workers choose the same level of effort*, so that $e_j = e_k = e^*$. When $e_j = e_k$ the first-order condition becomes

$$(W_1 - W_2)g(0) = \delta e^*$$

Using the fact that the distribution is uniform, so that $g(0) = \frac{1}{2b}$, we obtain the key condition

$$\frac{(W_1 - W_2)}{2b} = \delta e^* \qquad \text{(ICC)}$$

This condition, which determined the optimum amount of effort, acts as an incentive compatibility constraint. From this condition, two important conclusions follow:

Effort is larger the larger the wage spread.

Indeed an increase in the difference $(W_1 - W_2)$ yields an increase in effort.

The more important luck is in determining the outcome, the lower is effort.

To see this recall that the variance of z is $Var(z) = \frac{b^2}{3}$. So the larger b is, the larger is the variance of the luck component. This makes a lot of sense. If most of your output depends on how lucky you are, why should you bother to exercise effort? In production environments where measurements of effort are noisy, large rises must be given in order to offset the tendency by workers to reduce effort.

The worker has also a participation constraint. Note that under the symmetrical Nash equilibrium, each worker has a probability of winning the match equal to $G(0) = 1/2$. This implies that, in equilibrium, a worker's expected utility is

$$U = \frac{W_1 + W_2}{2} - \frac{\delta e^{*2}}{2}$$

and the worker will participate in the match (assuming that the outside option is 0) as long as expected utility is positive, so that

$$\frac{W_1 + W_2}{2} \geq \frac{\delta e^{*2}}{2} \qquad \text{(Participation Constraint)}$$

8.2.2 FIRM PROBLEM

The firm wants to maximize expected profit, or equivalently, profit per worker, since the number of workers hired is assumed exogenous to this problem. Expected output per worker is equal to $E(q) = e^*$ while the average cost of each worker is simply the average prize. This implies that the expected profits of the firm are

$$E[\Pi] = e^* - \frac{W_1 + W_2}{2}$$

The firm tries to maximize profits by choosing W_1 and W_2 subject to the participation constraint. Obviously the firm will set wages low enough to have

the participation constraint satisfied with equality. In other words, the firm problem is

$$Max_{W_1,W_2} \qquad e^* - \frac{W_1 + W_2}{2}$$

$$\text{s.t.} \quad \frac{W_1 + W_2}{2} = \frac{\delta e^{*2}}{2}$$

which becomes simply

$$\Pi_{W_1,W_2} = e^* - \frac{\delta e^{*2}}{2}$$

So that the first-order condition is

$$\frac{\partial \Pi}{\partial W_1} = \frac{\partial e^*}{\partial W_1} - \delta e^* \frac{\partial e^*}{\partial W_1} = 0$$
$$= \frac{1}{2b\delta}\left[1 - \frac{e^*}{\delta}\right]$$

where we used the fact that $\frac{\partial e^*}{\partial W_1} = \frac{1}{2b\delta}$ from the ICC, which suggests that when wages are chosen optimally

$$e^* = \frac{1}{\delta} \qquad \text{(efficient effort)}$$

A conclusion immediately follows:

Tournaments elicit the optimal amount of effort.

There are two ways to see this conclusion. First, firms force workers to induce effort up to the point in which the marginal cost of effort (namely δe^*) is equal to the marginal benefit (which is 1). Second, note that this condition is the same condition that appeared in Chapter 6 on optimal wage contracts when $p = 1$. Indeed, with a risk-neutral worker, you recall that the optimal contract requires $\beta = 1$ so that optimal effort is equal to $e^* = \frac{1}{\delta}$. This is the same result that we have now obtained. In other words, a tournament is an optimal compensation package, and satisfies all the conditions of the optimal bonus scheme analysed in Chapter 6 for a risk-neutral worker.

8.2.3 OBTAINING THE WAGES

To obtain the wages, one needs to substitute the optimal level of effort into the incentive compatibility constraint and in the participation constraint, so as to obtain a linear system of two equations in two unknowns. In other words, the system to solve is

$$\frac{W_1 + W_2}{2} = \frac{\delta e^{*2}}{2}$$
$$\frac{(W_1 - W_2)}{2b} = \delta e^*$$

where the optimal level of effort is $e^* = \frac{1}{\delta}$. Using this result we get

$$\frac{W_1 + W_2}{2} = \frac{\delta 1}{\delta^2 2}$$
$$\frac{(W_1 - W_2)}{2b} = 1$$

so that

$$(W_1 + W_2) = \frac{1}{\delta}$$
$$W_1 - W_2 = 2b$$

which gives the final wages and completes the problem

$$W_1 = b + \frac{1}{2\delta}$$
$$W_2 = \frac{1}{2\delta} - b$$

8.2.4 HETEROGENEITY AND RISKY STRATEGIES

If players are not identical there is a problem with tournament, since it becomes very difficult to elicit effort from workers. Because of the natural advantage that more able players possess, both individuals will not work hard enough. The more able worker will tend to shirk since he is likely to win anyway, the less able because he is likely to lose anyway. There is a way to solve this problem. It is called a *handicapping* system. Such a system gives the less able player a head start, so that makes it harder for the more able player to win.

Another feature of tournament with heterogeneous workers is linked to the possibility of choosing strategies that affect not only the average probability of winning, but also its variance. Imagine that two players can choose between two strategies (call the two strategies safe and risky) that offer them the same average probability of winning but have different variance. One can show that with heterogeneous contest, the better individuals will avoid high-risk actions. Choosing the risky actions (high variance) increases the chances of winning the context but also makes it more risky. Since better players are likely to win anyway and winning by little or by a lot makes little difference, this option offers little gain. As a result, more able players will play it safe and choose a low-variance action. The reverse is true for less able players. Since losing by a little or a lot is still losing and this is the likely outcome, expanding the negative tail of the performance distribution has little cost. Less able players should take a chance. Consequently, where tournaments encompass players of unequal ability, *there should be a negative relation between ability and variance of performance.*

8.3 Linear Performance Evaluation

To properly understand the role of tournaments and its specificity, particularly with respect to the case studies we will examine, it is important to introduce the concept of Linear Performance Evaluation (LPE). Under LPE players are rewarded on the basis of relative performance, but there are some important differences between a pure tournament and LPE. Suppose that there are many workers and assume that the salary structure is obtained by the sum of two components: a fixed part plus a bonus (penalty) term, with the bonus term linked to the average performance of the workers. If an individual worker produces and performs better than the average, he or she receives an increase in compensation equal to m times the difference between his performance and the average. In formula, such a reward scheme for individual i reads

$$W_i = W + m(e_i - Q) - \frac{\delta e_i^2}{2}$$

where W is the contractually specified reward to a player with average performance, m is the incremental reward (penalty) for above (below) average performance, and Q is the average performance of all players. If individual i views the average performance Q as fixed, he will choose e_i such that

$$m = \delta e_i$$
$$e_i = \frac{m}{\delta}$$

Notice that under LPE effort, and so performance, q_i, depends positively on the incremental reward for better relative performance, m, but is unaffected by the average level of rewards, W. This feature holds for both LPE and tournaments. In dimension LPE looks very much like a tournament.

The key difference between LPE and pure tournament comes about when heterogeneity is relevant. The thing to realize is that in LPE winning by a lot is important, while in pure tournaments all that is important is winning. This