

Tournaments

Class 8 – July 29, 2010

Prerequisite: Appendix D and E

Textbook: Lazear (25-37), Garibaldi (132-8)

Articles: Becker and Huselid (336-344)

Introduction

- Incentive contracts are often about **individual** performance
- However, agents typically interact with each other
- Depending on the type of interaction, we have:
 - **Tournaments** – agents compete against each other
 - **Teams** – agents co-operate with each other

Outline

1. Tournaments: Main Ideas
2. Tournament Model
3. Empirical Strategy: Fixed Effects
4. Application: NASCAR
5. Advantages and Disadvantages of Tournaments

1. Tournaments: Main Ideas

- How to address the incentive problem?
- Two or more agents compete
- Winner obtains higher prize than loser
- Winning depends on agents' effort

Incentives
to work
hard to win

Examples

- Promotion in firms
- Academic scholarships
- Professional sports

Tournaments and Soccer

- May 18, 1977 – last day of soccer season
- Only two teams from Sunderland, Bristol, or Coventry stay in the First Division of the English Football League
- Two games:
 - Game 1: Sunderland vs. Club X
 - Game 2: Bristol vs. Coventry
- Bristol-Coventry Game
 - Starts five minutes later than Sunderland game
 - Until minute 80: “fast and furious”, “fierce”, score 2:2
 - **Minute 80: players learn Sunderland lost**
 - After 80: “players pushed the ball around with little or no challenge”, “players on opposite teams joined in celebration”

2. Tournament Model

- Two risk-neutral and identical agents

(1) $q_i = e_i + u_i$ with $E[u_i] = 0$ for agent $i = 1, 2$

(2) $c(e_i) = 0.5e_i^2$ for agent $i = 1, 2$

⇒ Efficient level of effort: $e_1 = e_2 = 1$

Can tournaments achieve this outcome???

Tournament Payment Structure

- The winner receives W , the loser gets w
- The winner is the agent with better performance
 - E.g. Agent 1 wins if $q_1 > q_2$, Agent 2 wins if $q_2 > q_1$.

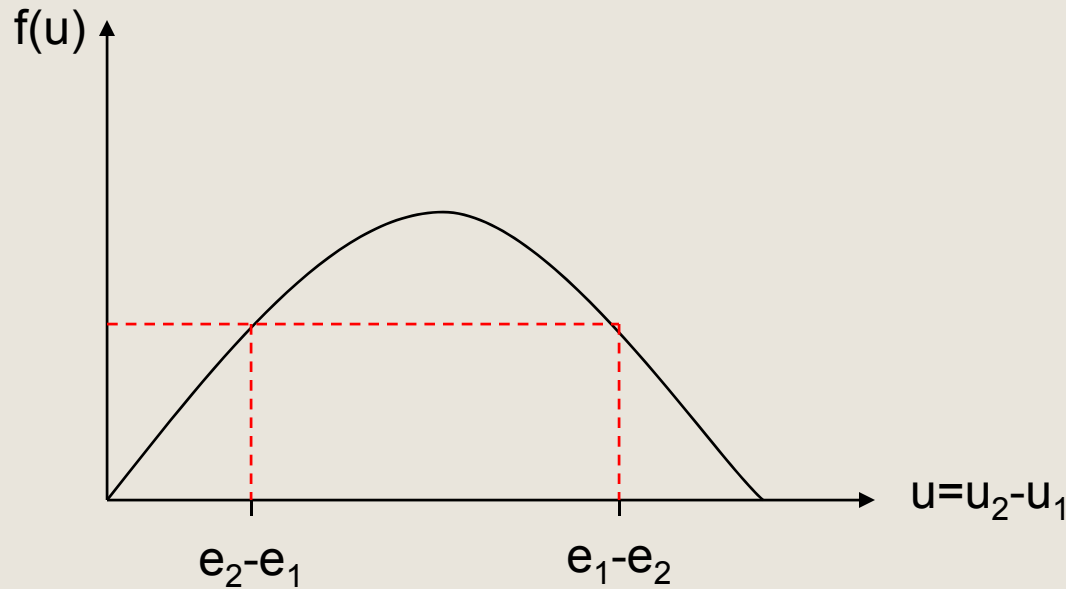
Agents' Payoffs

Agent 1

- $E[U_1] = pW + (1-p)w - c_1 = w + p(W-w) - 0.5e_1^2$
- $p = \text{Prob}(q_1 > q_2) = \text{Prob}(e_1 + u_1 > e_2 + u_2) = \text{Prob}(u_1 - u_2 > e_2 - e_1)$
 $= \text{Prob}(u > e_2 - e_1) = 1 - F(e_2 - e_1)$
 - where $u \equiv u_1 - u_2$ is a random variable with cumulative distribution function F and *symmetric* density function f
- Recall that $dF(u)/du = f(u)$

Symmetric Density Functions

- Many density functions f are symmetric: $f(x)=f(-x)$
- Examples: uniform, normal
- Here, it implies that $f(e_2-e_1)=f(e_1-e_2)$



2. Tournament Model

Agents' Payoffs – Contd.

Agent 1

$$\begin{aligned} E[U_1] &= pW + (1-p)w - c_1 \\ &= w + p(W-w) - 0.5e_1^2 \\ &= w + [1 - F(e_2 - e_1)](W-w) - 0.5e_1^2 \end{aligned}$$

Agent 2

$$\begin{aligned} E[U_2] &= pW + (1-p)w - c_2 \\ &= w + p(W-w) - 0.5e_2^2 \\ &= w + [1 - F(e_1 - e_2)](W-w) - 0.5e_2^2 \end{aligned}$$

Incentive Compatibility Constraints

$$\text{Max } E[U_1] = w + [1 - F(e_2 - e_1)](W - w) - 0.5e_1^2$$

- First-Order Conditions:

$$\text{Agent 1: } f(e_2 - e_1)(W - w) = e_1$$

$$\text{Agent 2: } f(e_1 - e_2)(W - w) = e_2$$

- If f is *symmetric*, we have that $f(e_2 - e_1) = f(e_1 - e_2)$

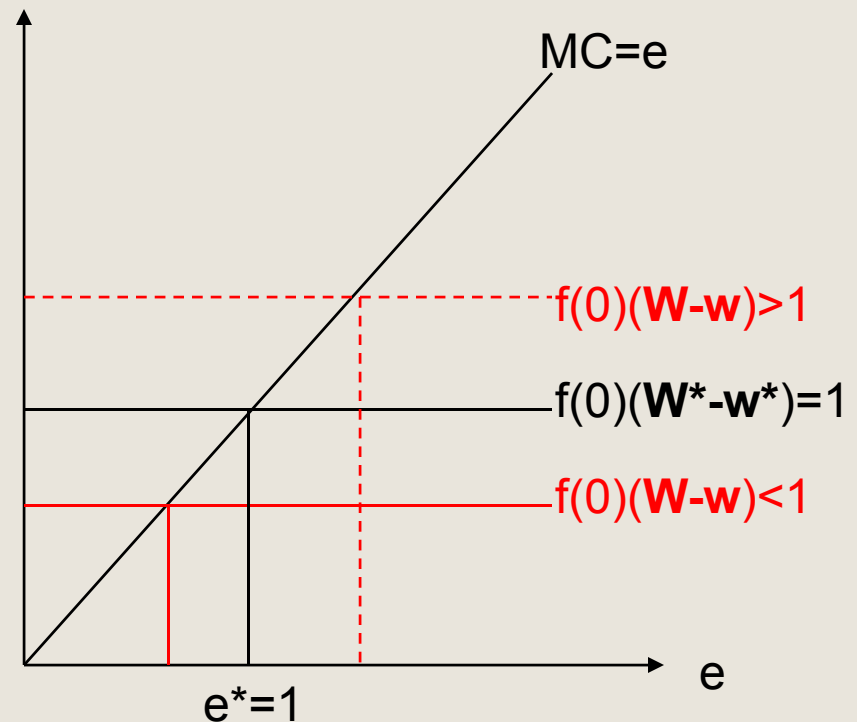
$$\Rightarrow e_1 = e_2 = e$$

$$\Rightarrow f(e_2 - e_1) = f(e_1 - e_2) = f(0)$$

$$e = f(0)(W - w)$$

What does it mean? Optimal Prize Spread

- Marginal cost of effort is e .
- Social marginal benefit of effort is 1.
- Private marginal benefit of effort is $f(0)[W-w]$.
- Optimal prize spread $W^* - w^*$ that equates social and private marginal benefit of effort is $1/f(0)$.



Example: Uniform Distribution

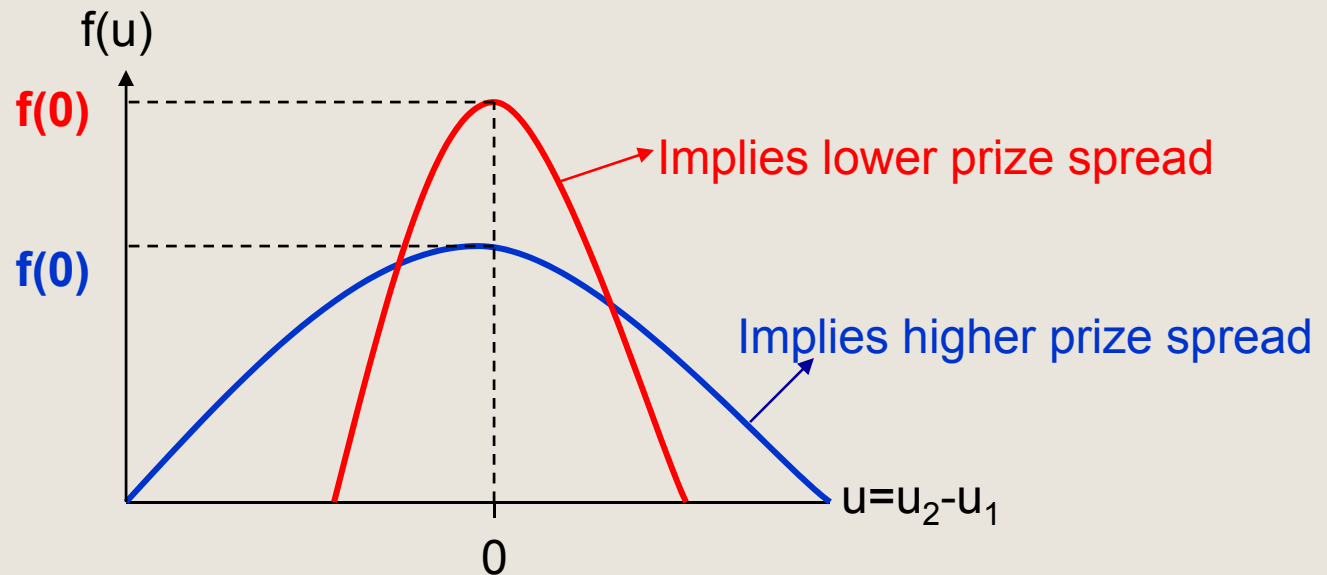
- Assume:
 - $q_i = e_i + u_i$ and $c_i = 0.5e_i^2$
 - $u \equiv u_1 - u_2$ has a uniform distribution on interval $[-5, 5]$

What is the optimal prize spread $W^* - w^*$?

- For a uniform distribution, $f(x) = 1/(b-a)$, and $F(x) = (x+b)/(b-a)$, where b is the end point and a is the start point
- Here, $f(0) = 1/(b-a) = 1/(5 - (-5)) = 1/10$
- Therefore, $W^* - w^* = 1/f(0) = 10$.
- Each equally likely to win: $F(0) = (0+5)/(5 - (-5)) = 0.5$

What does it mean? Importance of Luck

- Optimal prize spread $W^* - w^*$ varies negatively with $f(0)$
- $f(0)$ represents the importance of luck in winning:
 - low $f(0)$ means that luck is relatively important
 - high $f(0)$ means that luck is relatively unimportant



Example: Normal Distribution

- Assume:
 - $q_i = e_i + u_i$ and $c_i = e_i^2/2$
 - $u \equiv u_1 - u_2$ has a normal distribution with mean 0 and variance σ^2

What is the optimal prize spread $W^* - w^*$?

- With normal distribution, $f(x) = \exp[-(x-0)^2/2\sigma^2]/(\sigma(2\pi)^{0.5})$
- Here, $f(0) = 1/\sigma(2\pi)^{0.5} \simeq 1/2.5\sigma$
- Therefore, $W^* - w^* = 1/f(0) \simeq 2.5\sigma$

Tournament Prizes

Participation Constraint

$$\begin{aligned} E[U] &= w + p(W-w) - 0.5e_1^2 = R \\ &= w + 0.5(W-w) - 0.5 = R \end{aligned} \quad (\text{since } p=0.5, e=1)$$

System of Two Equations

(PC) $W+w=1+2R$

(ICC) $1=f(0)(W-w)$

- Solving for W and w yields:

$$W^* = 0.5 + R + 0.5/f(0)$$

$$w^* = 0.5 + R - 0.5/f(0)$$

Optimal Tournament Contract

- Suppose:
 - The principal cannot observe or verify agents' effort;
 - The agents and principal are risk neutral;
 - Agents are identical: $q_i = e_i + u_i$ and $c_i = e_i^2/2$, for agents $i=1,2$;
 - The winner of the tournament is the agent with higher q .
- Then, the prize spread that induces the optimal level of effort $e^*=1$ is $W^* - w^* = 1/f(0)$
 - where W denotes the prize for the winner, w denotes that prize of the loser, and f is the density function of a random variable $u_2 - u_1$

3. Empirical Strategy: Fixed Effects

- Main empirical implication of tournament theory:
 - $e=f(0)(W-w)$
⇒ Effort is higher when prize spread is higher.
- The challenge is to separate incentive from selection effects of tournaments.
- It's more exciting to watch Stanley Cup playoff games than regular season games.
 - There is more at stake in the playoffs so players try harder (incentive effect of higher prize spread),
 - Only good teams qualify for the playoffs (selection effect).

Cross-Section Comparison (variation by i)

- The potential outcomes are:

$$\left. \begin{array}{l} y_i(0)=u_i \\ y_i(1)=b+u_i \end{array} \right\} \text{Treatment effect} = b$$

- Naïve comparison yields:

$$E[y_i|D_i=1]-E[y_i|D_i=0] = b+ E[u_i|D_i=1]-E[u_i|D_i=0]$$

- Are treatment and control individuals similar?

3. Fixed Effects

Time-Series Comparison (variation by t)

- The potential outcomes are:

$$\left. \begin{array}{l} y_t(0) = u_t \\ y_t(1) = b + u_t \end{array} \right\} \text{Treatment effect} = b$$

- Naïve comparison yields:

$$E[y_t | D_t = 1] - E[y_t | D_t = 0] = b + E[u_t | D_t = 1] - E[u_t | D_t = 0]$$

- Has anything else changed over time that would affect y?

Panel Comparison (variation by i and t)

- The potential outcomes are:

$$\left. \begin{array}{l} y_{it}(0) = u_{it} \\ y_{it}(1) = b + u_{it} \end{array} \right\} \text{Treatment effect} = b$$

- Naïve comparison yields:

$$E[y_{it}|D_{it}=1] - E[y_{it}|D_{it}=0] = b + E[u_{it}|D_{it}=1] - E[u_{it}|D_{it}=0]$$

- Are treatment and control individuals similar?
- Has anything else changed over time that would affect y ?

Fixed Effects Model

- Suppose unobserved factors (u_{it}) can be decomposed into those that vary over time (ε_{it}) and those that don't vary over time (α_i):

$$u_{it} = \alpha_i + \varepsilon_{it}$$

- Suppose also that factors that vary over time change similarly for the treatment and control groups

$$E[\Delta\varepsilon_{it} | T] = E[\Delta\varepsilon_{it} | C]$$

- Then, observed changes in outcomes between treatment and control groups identify the treatment effect

$$E[\Delta y_{it} | T] - E[\Delta y_{it} | C] = \beta$$

3. Fixed Effects

Example

$$y_{it}(0) = \text{ability} + e_{it}$$

$$y_{it}(1) = \text{ability} + 5 + e_{it}$$

$$e_{i1} = 0, e_{i2} = 4$$

Time	Person	Ability	Treatment	Outcome
1	A	2	0	2
1	B	4	0	4
2	A	2	0	6
2	B	4	1	13

Cross-Section: $13 - 6 = 7$

Time-Series: $13 - 4 = 9$

Fixed Effects: $(13 - 4) - (6 - 2) = 5$

4. Application: Becker and Huselid (1992)

Sample

- 28 NASCAR races in 1990
- 44 drivers who competed in at least 5 races

Model

$$y_{it} = \alpha_i + bD_{it} + cX_{it} + \varepsilon_{it}$$

- y = performance measure
- α = drivers' fixed effects
- D = prize spread
- X = control variables (correlated with both y and D)

BH(1992): Definition of Variables

Variable	Type	Definition
Adjusted Finish	Performance (Y)	Finish position, adjusted for speed of the race
Spread	Main independent variable (D)	Average prize for first n positions relative to last m positions
Start Position	Control (X)	Starting position
Lap Length	Control (X)	Length of lap
Caution Flags	Control (X)	Number of caution laps

BH (1992): Results

Dependent Variable = Adjusted Finish	
Variables	Estimate (St. Error)
Spread	-1.1187 (0.1965)
Cautious Flag	0.1676 (0.0419)
Start Position	0.2247 (0.0702)
Lap Length	0.7742 (0.9807)

5. Advantages and Disadvantages of Tournaments

- + Improve incentives for all agents in tournament.
- + Serve a complementary function of ranking agents.
- + Lower measurement costs (relative, not absolute)
- + May filter out risk common to all agents.
- + Useful with frequent technological changes.
- May induce agents to sabotage each other.
- May induce agents to collude against the principal.
- Agents may select which tournament to participate.
- No incentives to co-operate with other agents.