Multiple Signals and Tasks

Class 6 - July 22, 2010
(corrected version)

Textbook: Milgrom and Roberts (214-221)
Readings: Gibbons and Murphy (36-42), Hannaway (all)

Introduction

- When there is an incentive problem:
  - Piece rate of b=1 is optimal if the agent is risk-neutral
  - Piece rate of 0<b<1 is optimal if the agent is risk-averse

- How does the optimal piece rate contract change if:

1. Additional signals of performance are available?
2. Performance depends on several tasks?

Outline

Multiple Signals

1. Case Study: CEO Compensation
2. When to Use Additional Signals of Performance?
3. Application: Gibbons and Murphy (1990)

Multiple Tasks

4. Case Study: Teacher Compensation
5. Model of Multitasking
6. Application: Hannaway (1992)

1. Case Study: CEO Compensation

- How to provide incentives to risk-averse hospital CEOs to reduce wait times for cataract surgeries?

- Elements of Compensation:
  - Competitive base salary (a)
  - Bonus tied to performance (bq)
  - What if another measure of performance is available?
    - Wait time at other hospitals
    - Patient satisfaction
    - Mortality rate of patients

2. When and How to Use Additional Signals?

Compensation

$$w=a+bq+\theta s$$

where:

- $q=e+u$
- $E[u]=0$, $Var[u]=V$
- $E[s]=0$, $Var[s]=1$
- $Cov(q,s)=Cov(u,s)=\rho$

Questions

1. When should we use signal s?
   - $\theta=0$ or $\theta\neq0$?

2. How should we use signal s?
   - $\theta>0$ or $\theta<0$?

Main Ideas

- $E[w]=E[a+bq+\theta s]=a+be$
- $Var[w]=Var[a+bq+\theta s]= b^2V+\theta^2+2b\theta\rho$

- Using signal s affects only variation in payment (risk)
- Potential to use signal s to reduce risk
- Reduction in risk can improve incentives

Example: Number of midterms and their weight

One or two midterms?

- One midterm: less work, more risk
- Two midterms: more work, less risk

⇒ Multiple performance measures may reduce risk.

How to weigh midterms?

- Equal weights?
- Better midterm counts as ¾, worse midterm as ¼?

⇒ Weights on performance measures related to risk.

Optimal Weight on Signal *s*

- How to choose signal *s* to minimize risk

$$\text{Min}_{\theta}\ \text{Var}[w] = \text{Var}[a+bq+\theta s]$$
$$= b^2V+\theta^2+2b\theta\rho$$

- First-order Condition

  $2\theta+2b\rho=0$

  $\Rightarrow$ $\theta^*=-\rho b$

Interpretation: Weight on Signal *s*

$$\theta^* = -\rho b^*$$

- Use signal *s* only if $\rho \neq 0$
  - i.e. signal is informative about performance q
- Sign of θ^* opposite of ρ
  - If $\rho<0$, $\theta^*>0$
  - If $\rho>0$, $\theta^*<0$

Example: Bell-Curving Midterms

- Should midterms be bell-curved?
  - Let s be the class average
  - Let q be your raw grade
  - s and q are positively correlated ($\rho>0$)
- Your final grade:
  - If s is low ($s<60$), your grade is $q+5$
  - If s is high ($s\geq 60$), your grade is q

  } $\theta < 0$

- Example of relative performance evaluation

3. Application: Relative CEO Compensation

- Gibbons and Murphy (1990)

Sample

- About 2,000 CEOs from about 1,300 firms, 1974 to 1986

Definition of variables

- w = CEO salary and bonus
- q = firm's rate of return
  - continuously accrued rate of return received by shareholders, including price appreciation and dividends
- s = market return
  - Continuously accrued rate of return on the value-weighted portfolio of firms in the market

Theoretical Predictions

- $w=a+bq+\theta s$
  - $b>0$
  - $\theta<0$ (since $\rho>0$)

Results

- 10% increase in the firm's rate of return increases the average CEO's pay by about 1.6% ($b>0$)
- CEO's pay is negatively related to the market return ($\theta<0$)
  - $q=20\%, s=0\% \Rightarrow$ CEO pay increases by 9.0%
  - $q=20\%, s=20\% \Rightarrow$ CEO pay increases by 7.5%
  - $q=0\%, s=20\% \Rightarrow$ CEO pay increases by 4.0%

Optimal Piece Rate Contract with Multiple Signals

- Suppose:
  - The principal cannot observe or verify the agent's effort;
  - The agent is risk averse and the principal is risk neutral.
  - An additional signal of performance is available.
- Then,

1. The signal should be used only if correlated with performance.
2. The weight on the signal should be positive if the signal is negatively correlated with performance, and vice versa.

4. Case Study: Teacher Compensation

- Should teacher compensation be tied to performance of students on standardized tests?

Argument For

- Incentives to improve teaching efforts.

Argument Against

- Teachers may neglect topics not covered on tests (Multitasking).

5. Multitasking Model

Elements

- Parties (School Principal and Teacher)
- Production Function
- Contract
- Payoffs
- Information
- Timing

Production and Payment

Production Function

(1) $q=e_1+e_2$

- The principal cannot observe either e_1 or e_2
- The principal observes two signals on each task:

(2a) $y_1=e_1+u_1$, where $E[u_1]=0$, $Var[u_1]=V_1$

(2b) $y_2=e_2+u_2$, where $E[u_2]=0$, $Var[u_2]=V_2$

Payment

(3) $w=a+b_1y_1+b_2y_2$

Payoffs

School Principal

- Risk-neutral

(4) $E[\Pi]=E[q-w]=(1-b_1)e_1+(1-b_2)e_2-a$

Teacher

- Risk-averse

(4) $E[U]= E[w]-c(e)-0.5rVar[w]$

$= a+b_1e_1+b_2e_2-0.5(e_1+e_2)^2- 0.5r(b^2_1V_1+b^2_2V_2)$

Tasks substitutes

Signals uncorrelated

Payoffs

School Principal

- Risk-neutral

(4) $E[\Pi]=E[q-w]=(1-b_1)e_1+(1-b_2)e_2-a$

Teacher

- Risk-averse

(4) $E[U]= E[w]-c(e)-0.5rVar[w]$

$= a+b_1e_1+b_2e_2-0.5(e_1+e_2)^2- 0.5r(b^2_1V_1+b^2_2V_2)$

Information and Timing

Information Structure

- Principal cannot observe or verify $e_1$or e_2

Timing

1. Principal designs the contract [a, b_1, b_2]
2. Teacher accepts or rejects the contract
3. If teacher accepts, she chooses e_1 and e_2
4. Production and payoffs

Constraints

- Teacher chooses e_1 and e_2 to maximize E[U]:

  $$a+b_1e_1+b_2e_2-0.5(e_1+e_2)^2-0.5r(b^2_1V_1+b^2_2V_2)$$

- First-order conditions:

  $b_1=e_1+e_2$, $b_2=e_1+e_2$

  ➢ **(ICC) $b_1=b_2\equiv b=e_1+e_2$**

- To participate, E[U]=R. Using ICC, we have:

  $E[U]=a+be-0.5e^2-0.5rb^2(V_1+V_2)$

  ➢ **(PC) $a=R+0.5b^2+0.5rb^2(V_1+V_2)-b^2$**

Designing Optimal Contract

- The problem is to maximize

$$E[\Pi]=E[q-w]=(1-b_1)e_1+(1-b_2)e_2-a$$

subject to PC and ICC

  - **$E[\Pi]=b-0.5b^2-0.5rb^2(V_1+V_2)-R$**

- First-order condition:

$1 – b^* - rb^*(V_1+V_2)=0$

  - **$b^*=1/[1+r(V_1+V_2)]$**

Interpretation of $b^*=1/[1+r(V_1+V_2)]$

1. Equal Compensation Principle

- Here, tasks are substitutes: $c(e_1+e_2)$
- When tasks are substitutes, marginal return to each task must be identical ($b_1=b_2$); otherwise, the agent will perform only the task with the higher marginal return.

2. Trade-off between Risk and Incentives

- Optimal piece rate b depends negatively on variance of both signals.
- When either signal is very imprecise, incentive pay should not be used for either signal.

Job Design

- Suppose $V_1 \to \infty$ and $V_2=0$.
- If tasks are substitutes, the principal should pay a salary (b=0). But with a salary contract, $e_1=e_2=0$!

- Can the principal do better?

- Redesign the job:
  - Job 1: imprecise signal, pay salary, $e_1=0$
  - Job 2: precise signal, incentive pay, $e_1=e^*$.

6. Application: Hannaway (1992)

- Divide teacher's job into two parts:

1. Basic skills teacher (e.g. math)
2. Higher-order skills teacher (e.g. critical thinking)

- Easier to measure basic skills

➢ Use pay for performance for the basic skills teachers only

Optimal Piece Rate Contract with Several Tasks

- Suppose:
  - The principal cannot observe or verify the agent's effort;
  - The agent is risk averse and the principal is risk neutral.
  - Agent's performance depends on several tasks.
- Then,

1. If tasks are substitutes in agent's utility, each task should be rewarded equally at the margin (Equal compensation principle).
2. Tasks that are measured imprecisely can be separated into distinct jobs. These jobs should not tie pay to performance.
3. Tasks that are measured precisely can be separated into jobs that tie pay to performance.