Piece Rates – Risk Aversion

Class 5 – July 20, 2010

Pre-requisite: Appendix C

Readings: Garibaldi (100-104)

Introduction

- A piece rate contract:
  - Simple to design and implement
  - Provides right incentives when effort cannot be observed or verified

- However, this contract transfers all risk to the agent
- **What if the agent is risk-averse?**

Outline

1. Case Study: Northern Specialists' Alternative Payment Plan (APP)
2. Trade-off between Insurance and Incentives
3. Piece Rate Model with Risk Averse Agents
   a. Model Description
   b. Optimal Contract
4. Application: Northern Specialists' APP

1. Case Study: Northern Specialists' APP

- Northern Ontario (north of French River)
- Significant variation in physicians' income
- Alternative Payment Plan
  - Designed to address insurance and incentives
- Two elements:
  a. Fixed income
     - 30% of specialists' average fee-for-service income
  b. Pro-rated fee-for-service income
     - 70% of the value of each service provided

2. Trade-off Between Insurance and Incentives

- Suppose:
  - q=e+u
  - E[u]=0, Var(u)=V

Full Insurance

- w=s (salary, b=0)
- Var(w)=0 $\Rightarrow$ Full insurance
- **But e=0 if e cannot be observed!**

Optimal Incentives

- w=a+q (piece rate, b=1)
- Optimal effort e=e*
- **But Var(w)=V $\Rightarrow$ No insurance!**

How to resolve trade-off between insurance and incentives?

3. Piece Rate Model with Risk Averse Agent

Elements

1. Parties
2. Production Function
3. Payment
4. Payoffs
5. Information
6. Timing

Production and Payment

Production Function

(1) $q=e+u$

(2) $E[u]=0$, $Var[u]=V$

Payment

(3) $w=a+bq$

where:

- a = fixed payment
- b = piece rate

Payoffs: Principal

- Principal is risk neutral
- Cares only about the expected value, not variance

- Principal's payoff:

(4) $E[\Pi]=E[q]-E[w]$

$=E[e+u]-E[a-b(e+u)]$

$=(1-b)e-a$

Payoffs: Agent

- Agent is risk-averse
  - Cares about both expected value and variance

- Agent's payoff:

(5) $U = E[w] - 0.5 \times r \times Var(w) - c(e)$

$= E[a+b(e+u)] - 0.5 \times r \times Var[a+b(e+u)] - 0.5e^2$

$= a + be - 0.5rb^2V - 0.5e^2$

- r is the (absolute) coefficient of risk aversion
- Expresses how much agent dislikes risk

Information and Timing

Information Structure

- Principal cannot observe e, or she cannot verify it to a third party

⇒The contract cannot stipulate e

⇒Principal can only decide on a and b

Timing

1. Principal designs the contract [a,b]
2. Agent accepts or rejects the contract
3. If agent accepts the contract, he chooses e
4. Production and payoffs

Piece Rate Model with Risk Averse Agent

Element	Summary
Parties	Principal, agent
Production Technology	q=e+u, with E[u]=0
Contract	Payment: w=a+bq Contract: [a, b]
Payoffs	Agent: $E[U]=a+be-0.5rb^2V-c(e)$ Principal: $E[\Pi]=(1-b)e-a$
Information	Principal cannot observe effort
Timing	1. Principal designs the contract 2. Agent accepts or rejects 3. Agent chooses effort 4. Payoffs

Designing Optimal Contract

The Problem

$\text{Max}_{a,b}\ E[\Pi] = (1-b)e-a$

subject to:

(PC) $U = a+be-0.5rb^2V-0.5e^2 \geq R$

(ICC) $e=\text{argmax}_e\ U$

- PC = participation constraint
- ICC = incentive compatibility constraint

Solution by Backward Induction

Step 1: Incentive Compatibility Constraint

- Choose e to Max $a+be-0.5rb^2V-0.5e^2$
- First-order condition:

(ICC) $b-e=0 \Rightarrow e=b$

Step 2: Participation Constraint

- (PC) $U= a+be-0.5rb^2V-0.5e^2 =R$

$\Rightarrow a=R+0.5e^2+0.5rb^2V-be$

Step 3: Objective Function

$E[\Pi]=(1-b)\mathbf{e}-\mathbf{a}$

- Substitute for **a** from PC:

$\Rightarrow E[\Pi]=(1-b)e-(R+0.5e^2+0.5rb^2V-be)$

$\Rightarrow E[\Pi]=e-0.5e^2-0.5rb^2V-R$

- Substitute for **e** from ICC:

$\Rightarrow E[\Pi]= b-0.5b^2-0.5rb^2V-R$

Step 4: Maximize $E[\Pi]$

$$\text{Max}_b \; E[\Pi] = b - 0.5b^2 - 0.5rb^2V - R$$

- First-order condition:

$1 - b^* - rb^*V = 0$

$\Rightarrow b^* = 1/(1+rV)$

- Substitute for a from PC:

$\Rightarrow a^* = R + 0.5b^{*2} + 0.5rVb^{*2} - b^*e$

Interpretation: Payment

$$a^{*} + b^{*}e = R + 0.5e^2 + 0.5rb^2V$$

- Payment compensates the agent for:
  1. Opportunity cost (R)
  2. Cost of effort ($0.5e^2$)
  3. Risk premium ($0.5rb^2V$)

Interpretation: Piece Rate

$b^* = 1/(1+rV)$

- Incentive part of compensation
  - Varies negatively with r and V

- $0 < b^* < 1$
  - Neither optimal insurance (b=0) or incentives (b=1)
  - Second-best solution

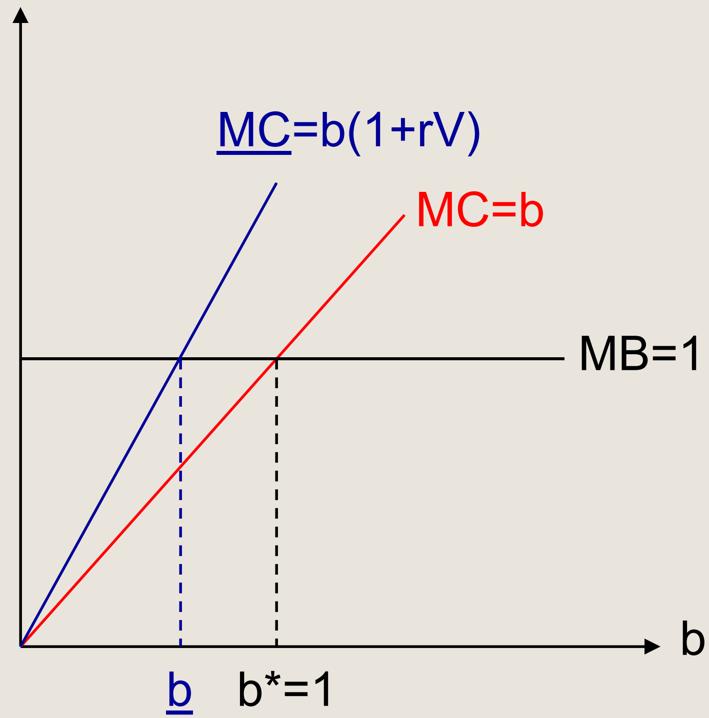
What does this mean?

<u>Contract with Risk-Neutral Agent</u>

- Max E[q]-c(e)=e-0.5e^2=b-0.5b^2
- (Remember that b=e from ICC)
  - MB(b)=1
  - MC(b)=b

<u>Contract with Risk-Averse Agent</u>

- Max E[q]-c(e)-RP=e-0.5e^2-0.5b^{2}rV =b-0.5b^2-0.5b^{2}rV
  - MB(b)=1
  - MC(b)=b+brV=b(1+rV)



4. Application: Northern Specialists' APP

Fee-for-Service Payment

- $w=a+bpq = pq$
- No base payment ($a=0$) and 100 percent piece rate ($b=1$)
- $q=e+u$, with $E[u]=0$
- $c(e)=2.5e^2$

Data

- Annual days of work (e) = 200 days
- Daily income (p) = \$1,000
- ➢ $E[w]=pe=\$1{,}000\times200=\$200{,}000$

Option 2: APP

Goals

- Provide some insurance to attract and retain physicians
- Ensure that physician works at least 180 days

Proposed Payment

- Base (a) is 30% of annual FFS salary, or: a = \$60,000
- Piece rate (b) is 0.7

➢ $E[w]=a+pbe=\$60,000+\$1,000\times0.7e=\$60,000+\$700e$

Will this contract achieve the stated goals?

Choice of Days under APP

$$\text{Max}_e\ E[U]=E[w]-c(e)-RP$$

$$=60{,}000+700e-2.5e^2-0.5b^2rV$$

First-order condition

- $700=5e$

$\Rightarrow e=140 < 180!$

$\Rightarrow$APP physicians work less than the minimum acceptable 180 days of work.

How to Solve Low Days of Work Problem?

Legal Solution

- If billings less than \$180,000, review physician practice ("10 percent corridor or buffer zone")
- Can result in significant monitoring costs

Economic Solution

- Days of work (e) $\geq$ 180
- First-order condition for choice of days of work:
  - $pb=5e \Rightarrow 1{,}000b=5(180)$

$\Rightarrow$ b=0.9

How much of FFS to convert?

- To be attractive, $U^{APP} \geq U^{FFS}$
- U^{FFS}=E[w]-c(e)-RP=a+bpe-2.5e^2-b^{2}rV
  $= 1{,}000 \times 200 - 2.5 \times 200^2 - rV$
  $= 100{,}000 - rV$
- U^{APP} =E[w]-c(e)-RP = a+pbe-2.5e^2-b^{2}rV
  $= a + 1{,}000 \times 0.9 \times 180 - 2.5 \times 180^2 - 0.9^2 rV$
  $= a + 81{,}000 - 0.81rV$

$\Rightarrow$ a $\geq$\$19,000-0.19rV

Revised APP Contract

- **Offer $20,000 plus 90% on FFS**
- Physicians voluntarily accept contract and work 180 days
- No need for monitoring and reviews

- What if risk aversion is not the only problem to retain and attract physicians to Northern Ontario? (e.g. weather, isolation)
- The solution is to adjust base (a), not piece rate (b)
  - Base may include any type of compensating premiums, such as risk premium, geographic modifier, etc.
  - Base affects whether the contract is acceptable or not, but has no effect on days of work
  - Changes in piece rate affect the provision of effort

Optimal Piece Rate Contract with Risk Averse Agent

- Suppose:
  - The principal cannot observe or verify the agent's effort;
  - The agent is risk averse and the principal is risk neutral.
- Then, the optimal piece rate contract [a*,b*] is such that:

1. 0 < b* <1
   - b* depends negatively on agent's risk aversion and variance in output
2. a*>0
   - a* compensates the agent for opportunity cost, cost of effort, and risk premium