Piece Rates - Theory

Class 3 - July 13, 2010

Readings: Lazear (13-19), Garibaldi (82-98)

Introduction

- When the agent's effort can be observed
- ➢ The optimal contract satisfies two conditions:
  1. Choice of effort balances costs and benefits
  2. Payment acceptable to both agent and principal

- What if the agent's effort cannot be observed?
- ➢ The contract cannot specify the effort level
- ➢ How can the principal then be sure that the agent will take the right effort level?

Outline

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2. Problems with Salary Contract
3. Piece Rate Contract
   a. Model Description
   b. Optimal Piece Rate Contract
4. Application: Piece Rate Contracts in Sales

1. Case Study: Economics of Driving Taxi Cabs

	Lease Drivers	Shift Drivers
Own Plate?	No	No
Owns Car?	Yes	No
Expenses		
Dispatch	Yes	Yes
Car Payments	Yes	
Insurance	Yes	
Maintenance	Yes	
Gas	Yes	Yes
Lease Fee	Yes	
Rental Fee		Yes
Income		
Fares and Tips	Yes	Yes
Rental Fee	Yes	

Contract between Lease and Shift Drivers

- Contract:
  - Shift driver pays a rental fee to lease driver
  - Shift driver keeps all fares and tips

- Puzzle
  1. Why not a salary contract?
     - E.g. pay shift driver salary for driving 8 hours/day, and lease driver keeps all fares and tips
  2. Why not share fares and tips?
     - E.g. Low or no rental fee, and share fares and tips

2. Salary Contract

- p = average fare per customer $\equiv 1$
- q = number of customers per day
- e = driver's effort

Optimal Salary Contract [e*,s*]

1. [e*]: $q'(e^*)=c'(e^*)$ Efficient level of effort
2. [s*]: $S^* \geq R+c(e^*)$ Participation

Salary Contract and Incentive Problem

1. e cannot be observed, or verified
   $\Rightarrow$ Contract cannot specify e, only s
2. $\Pi = q(e) - s \quad \Rightarrow \Pi_e > 0,\ \Pi_S < 0$
3. $U = s - c(e) \quad \Rightarrow U_e < 0,\ U_S > 0$

} Incentive Problem

- Choice of Effort by Shift Driver

$$\text{Max}_e\ U = s - c(e)$$

$$\Rightarrow \mathbf{e=0!}$$

Examples

- "Well, then, says I, what's the use you learning to do right when it's troublesome to do right and ain't no trouble to do wrong, and the wages is just the same?"

  Mark Twain, *Adventures of Huckleberry Finn*

- Denis Rancourt, a tenured physics professor at U of Ottawa, awarded A-pluses to all students in a senior physics course

Problem with Salary Contract

- When effort cannot be observed, the contract must address two problems:
  1. Induce participation
  2. Provide incentives

- Fixed payment (salary) addresses participation only
- We need something else to address incentives, something tied to effort level

Piece Rate Contract: Main Ideas

$$w=a+b\times q$$

where:

- w = income of shift driver
- a = fixed payment, independent of performance (q)
- $b\times q$ = variable payment, depends on performance (q)
- b is known as the piece rate and takes values of [0,1]

Contract	a	b
Salary	>0	0
Rental Fee, keep all fares	<0	1
Rental Fee, share fares	<0	0<b<1

What is optimal [a, b] when e cannot be observed?

Piece Rate Model: Elements

1. Parties
2. Production Function
3. Payment (Contract)
4. Payoffs
5. Information
6. Timing

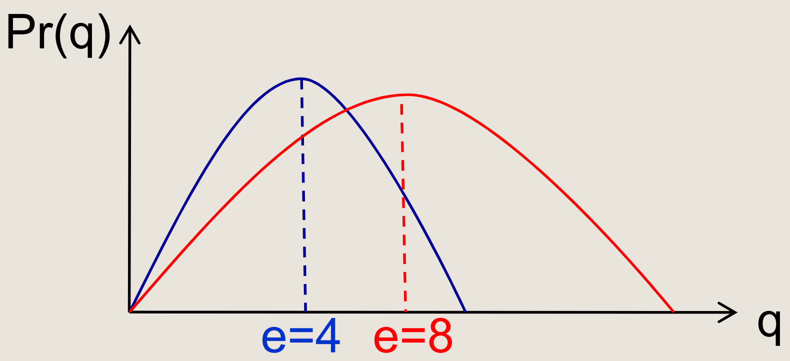
Piece Rate Model: **Production Function**

(1) $q=e+u$

- u are factors that shift driver cannot control (e.g. weather)
- u is a random variable, with mean of zero and variance V

(2) $E[u]=0$, $Var[u]=V$

$\Rightarrow E[q]=e$, $Var[q]=Var[u]=V$



Piece Rate Model: **Payment**

- The payment is:

(3) $w=a+bq$

- where:
  - a = fixed payment (can be positive or negative)
  - b = piece rate, ranges between zero and one

- Expected value of payment is:

(4) $E[w]=a+bE[q]=a+be$

Piece Rate Model: **Payoffs**

- Actual payoffs are:

(6) Π=q-w=q-a-bq=(1-b)q-a lease driver

(7) U=w-c(e)=a+bq-0.5e^2 shift driver

- Actual payoffs are random because q is random
- Preferences toward risk
  - Risk neutral: care about expected value only
  - Risk-averse: care about both expected value and variance
- Assume both lease and shift drivers are <u>risk-neutral</u>
- Then, their payoffs are:

(8) E[Π]=(1-b)E[q]-a=(1-b)e-a

(9) E[U]=a+bE[q]-c(e)=a+be-0.5e^2

Piece Rate Model: **Information and Timing**

Information Structure

- Lease driver cannot observe e, or she cannot verify it to a third party

⇒The contract cannot stipulate e

⇒Lease driver can only decide on a and b

Timing

1. Lease driver designs the contract [a,b]
2. Shift driver accepts or rejects the contract
3. If the shift driver accepts the contract, he chooses e
4. Production and payoffs

Piece Rate Model– Summary of Elements

Element	Summary
Parties	Shift driver, Lease driver
Production Technology	$q=e+u$, with $E[u]=0$
Contract	Payment: $w=a+bq$ Contract: [a, b]
Payoffs	Shift driver: $E[U]=a+be-c(e)$ Lease driver: $E[\Pi]=(1-b)e-a$
Information	Lease driver cannot observe effort
Timing	1. Lease driver designs the contract 2. Shift driver accepts or rejects 3. Shift driver chooses effort 4. Payoffs

Designing Optimal Piece Rate Contract

The Problem

$\text{Max}_{a,b}\ E[\Pi] = (1\text{-}b)e\text{-}a$

subject to:

(PC) $E[U] = a+be-0.5e^2 \geq R$

(ICC) $e=\text{argmax}_e\ E[u]$

- PC = participation constraint
- ICC = incentive compatibility constraint

Solution by Backward Induction

Step 1: Incentive Compatibility Constraint

- For any a and b, choose e to Max $E[U]=a+be-0.5e^2$
- First-order condition:

(ICC) $e=b$

Step 2: Participation Constraint

- (PC) $E[U]= a+be-0.5e^2=R$

$\Rightarrow a=R+0.5e^2-be$

Step 3: Objective Function

$E[\Pi]=(1-b)e-a$

- Substitute for **a** from PC

$\Rightarrow E[\Pi]=(1-b)e-R-0.5e^2+be$

$\Rightarrow E[\Pi]=e-0.5e^2-R$

- Substitute for **e** from ICC (e=b)

$\Rightarrow E[\Pi]=b-0.5b^2-R$

Step 4: Maximize $E[\Pi]$

$$\text{Max}_b \; E[\Pi] = b - 0.5b^2 - R$$

- First-order condition:

$1 - b^* = 0 \Rightarrow b^* = 1$

- From ICC, we have:

$\Rightarrow e^* = b^* = 1$

- Substitute for a from PC:

$\Rightarrow a^* = R + 0.5e^{*2} - b^*e^* = R - 0.5$

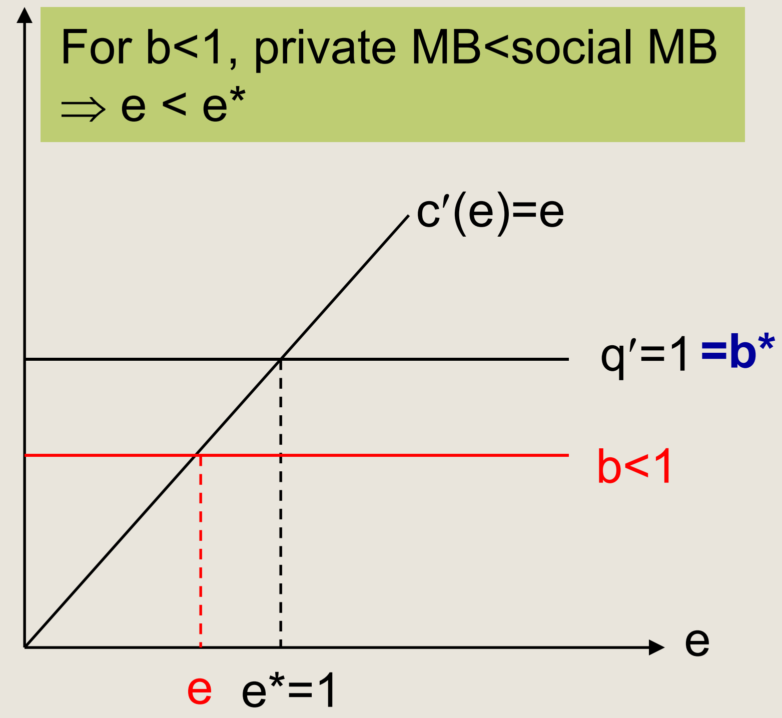
What does this mean?

Optimal piece rate

- $b^*=1 \Rightarrow e^*=1$
- In general, the optimal effort: $q'(e^*)=c'(e^*)$
- Given $E[q]=e$ and $c(e)=0.5e^2$, $q'(e)=1$ and $c'(e)=e$
- Therefore, $e^*=1$.

$\Rightarrow$ **Piece rate of $b^*=1$ induces efficient level of effort!**

For $b<1$, private MB<social MB $\Rightarrow e < e^*$



Optimal fixed payment

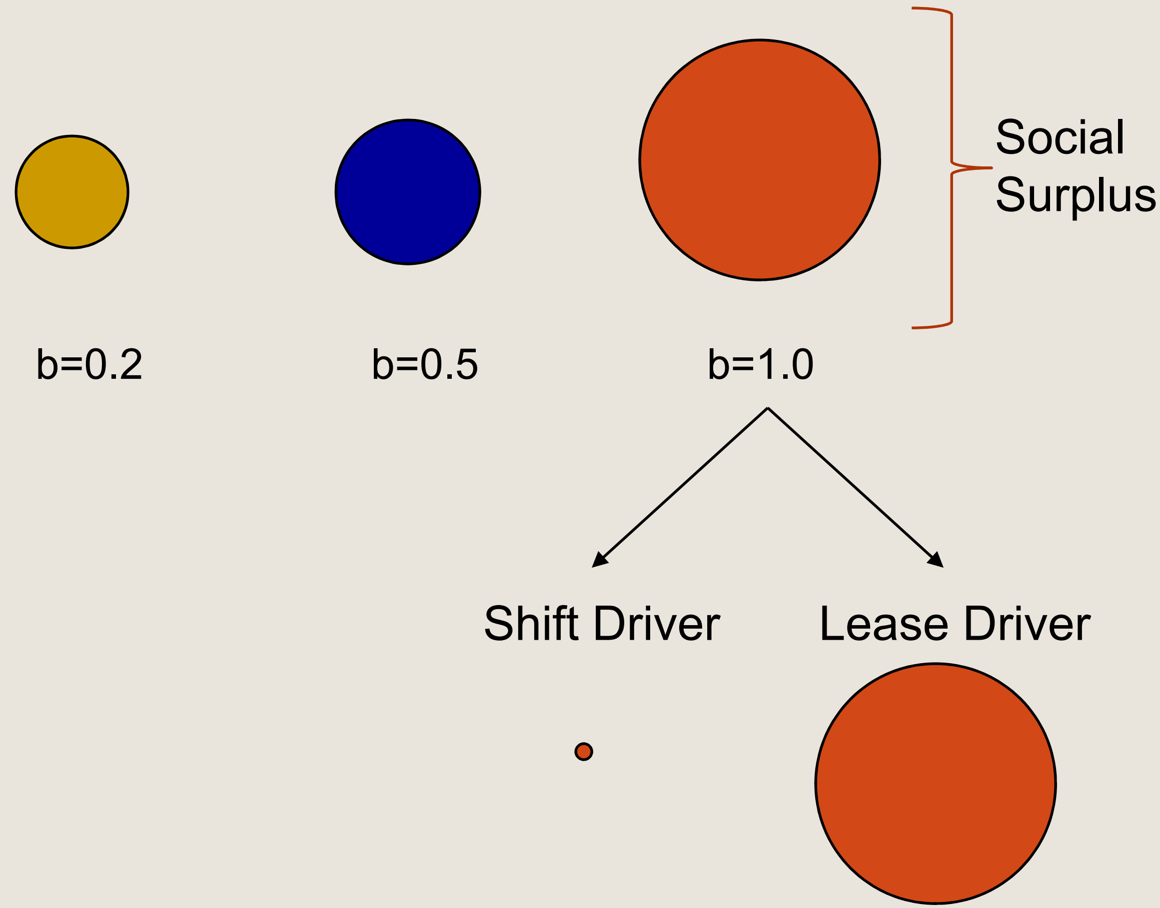
- $a=R+0.5e^2-be=R+c(e)-bE[q]$
- $-a^*=E[q^*]-c(e^*)-R$

- This is equal to social surplus
  - the net benefit the drivers earn in the relationship [E[q*]-c(e*)] relative to their outside options [R+0]
  - Lease driver extract all social surplus

Piece rate contract - intuition

Which b?

(Incentives)



Which a?

(Participation)

Application: Optimal Contract for Taxi Cabs

- Suppose:
  - $p=\$8$
  - $q=2e+u$, $E[u]=0$
  - $c(e)=\$e^2$
  - $R=\$6$
- What is the optimal effort and piece rate contract?

- $e^*=8$: $pE[q'(e^*)]=c'(e^*) \rightarrow 8*2=2e^* \rightarrow e^*=8$ hours
- $b^*=1$
- $a^*=-58$: $a^*=R+c(e)-pE[q]=\$6+\$8^2-8*2*8=-\$58$

4. Piece Rate Contracts in Sales

- Common to pay salesmen by commission:
  - Finance: e.g. mutual fund brokers
  - Insurance: e.g. life insurance policy brokers
  - Retail: e.g. percent of sales

- Two apparent difficulties with $b^*=1$ and $a^*<0$:
  1. Commission rate is rarely 100 percent
  2. Salesmen rarely pay a fixed amount to companies

Piece Rates and Commission Rates

- The commission rate is a percent on gross revenue, i.e. before fixed costs are paid off
- 10% on gross revenue can be close to 100% on net revenue
  - Gross revenue = pq(e,k), where k is capital
  - Net revenue = pq(e,k)-rk, where r is price of capital

  - Then, 1.0 $\times$net revenue=commission$\times$gross revenue

$\Rightarrow$ commission=net revenue/gross revenue

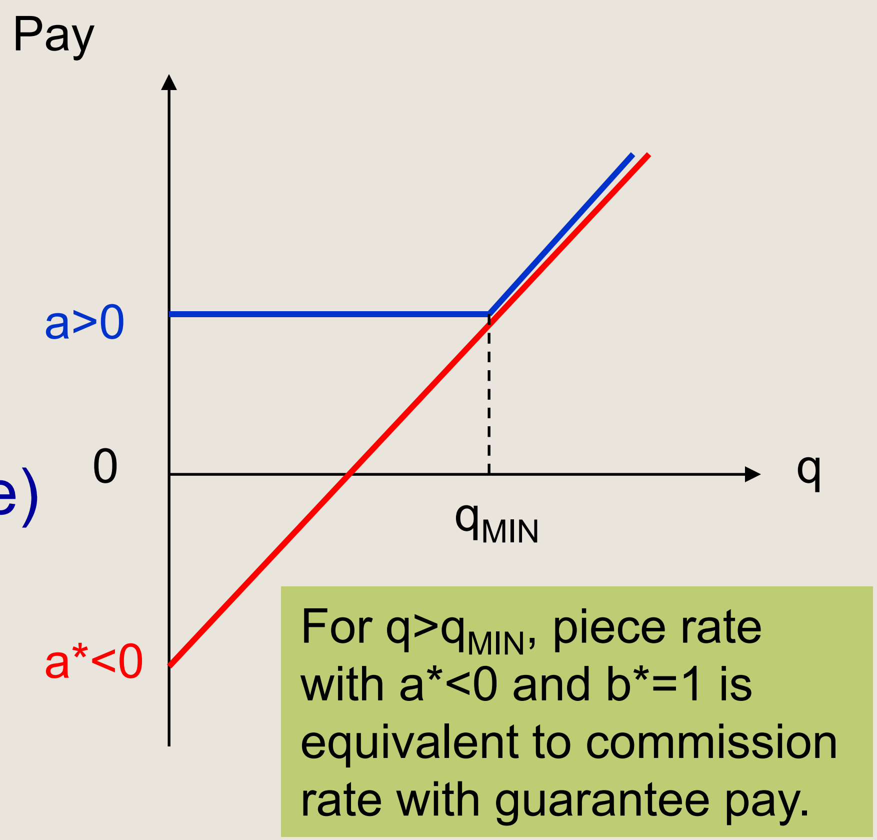
Negative Payment and Commissions

Piece Rates

- $b^*=1$, $a^*<0$

Commission Pay

- $b^*=1$
- $a>0$ (pay guarantee)
- Dismissal if q consistently below some level q_{MIN}



Optimal Piece Rate Contract with Unobservable Effort

- Suppose:
  - The principal cannot observe or verify the agent's effort;
  - The agent and principal are risk neutral.
- Then, the optimal piece rate contract [a*,b*] is such that:

1. $b^*=1$
2. $a^*<0$ satisfies the agent's participation constraint.

- This contract induces the efficient level of effort e*, defined by $pE[q'(e^*)]=c'(e^*)$.