

Salary Model

Class 2 - July 8, 2010

Pre-requisites: Appendix A

Introduction

- Employment relationships are governed by contracts, either explicit or implicit, which specify:
 - duties of employee (“work”)
 - payment obligations of employer
- Example: \$400/week for 5 days of work in a restaurant
 - How can we tell if this is the best contract? (Evaluation)
 - If it’s not, how can the contract be improved? (Design)

Outline

1. Case Study: Homeless and Mental Health
2. Salary Model
 - a. Model description
 - b. Optimal contract
3. Application: Salary Contract for Psychiatrists
4. Efficiency and Equity

1. Case Study: Homeless and Mental Health

- Over 30,000 homeless in Toronto
- The health care system often fails to provide adequate treatment for homeless people with mental illness or substance abuse.
- Assertive Community Treatment (ACT) teams:
 - A team of psychiatrists, nurses and social workers
 - Follows a small caseload of homeless mentally ill clients, seeking them out in the community to provide high-intensity mental health treatment and case management.
- The question is how to design the contract for psychiatrists:
 - How many days of work?
 - What is appropriate compensation?

2a. Salary Model - Description

How Would Economist Approach this Problem?

- Think of the Ministry as a firm wishing to employ a physician
⇒ Similar to profit-maximization problem
- The Ministry should design the contract, i.e. choose compensation and days of work:
 - a. To maximize its 'profits'
 - b. Yet make the contract acceptable to the physician

Ministry's 'Profits'

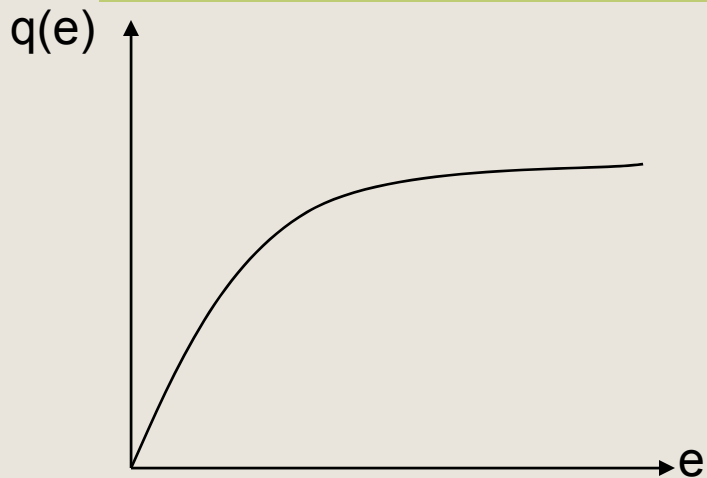
(1) $\Pi = pq - s$

- p is the 'price', e.g. the value of treating one patient
- q is the 'output', e.g. the number of patients treated
- s is the 'wage', e.g. physician's salary

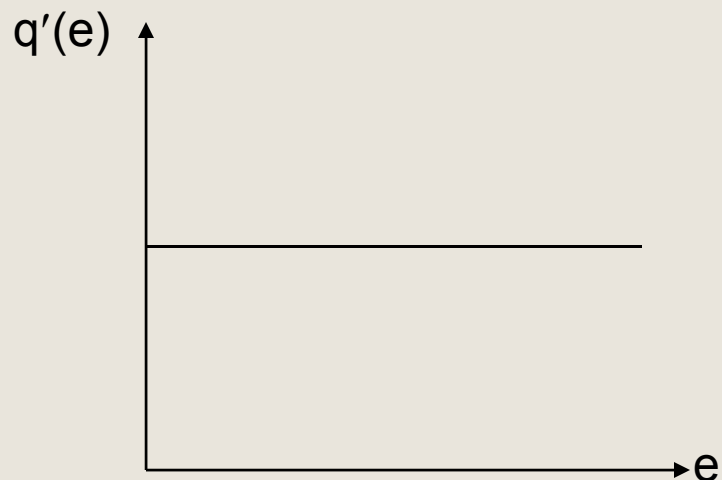
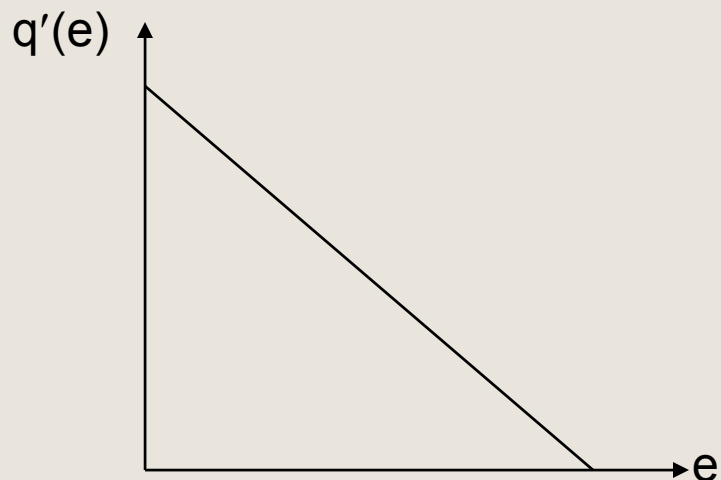
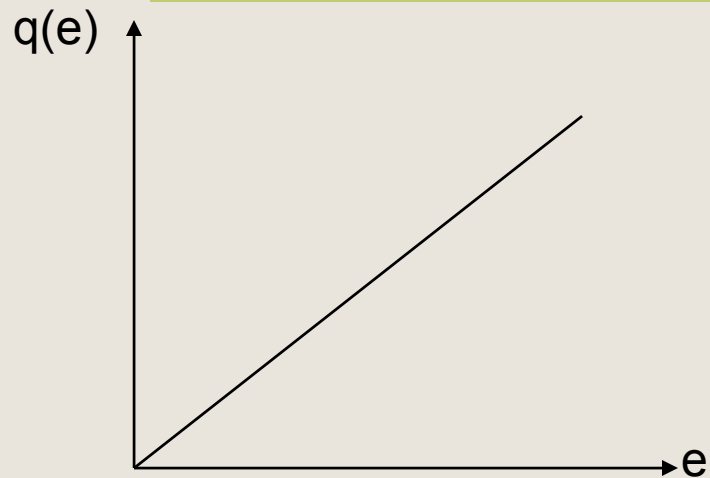
(2) $q = q(e)$

- e = days of work ('effort'), observed by Ministry
- $q(e)$ is the production function
- $q'(e) > 0$: marginal product
- $q''(e) \leq 0$: non-increasing marginal productivity

Case $q' > 0 > q''$, e.g. $q = \ln(e)$



Case $q' > 0 = q''$, e.g. $q = e$



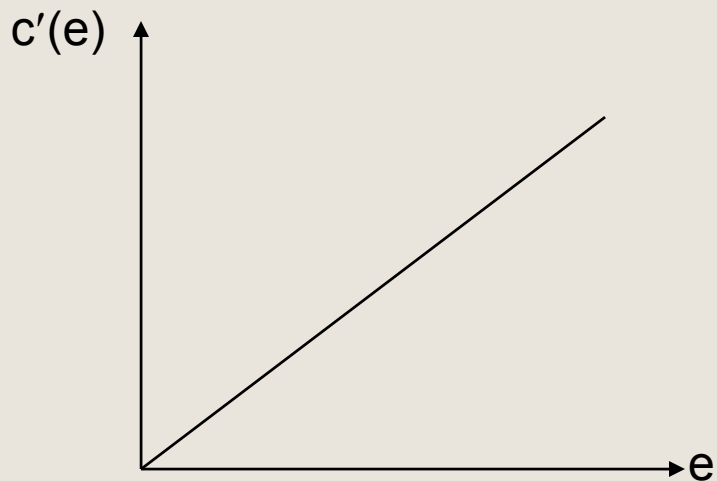
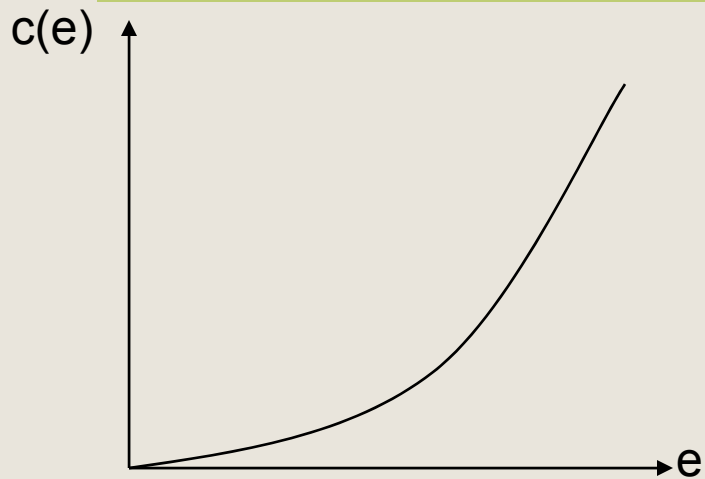
Physician Utility

$$(3) U = s - c(e)$$

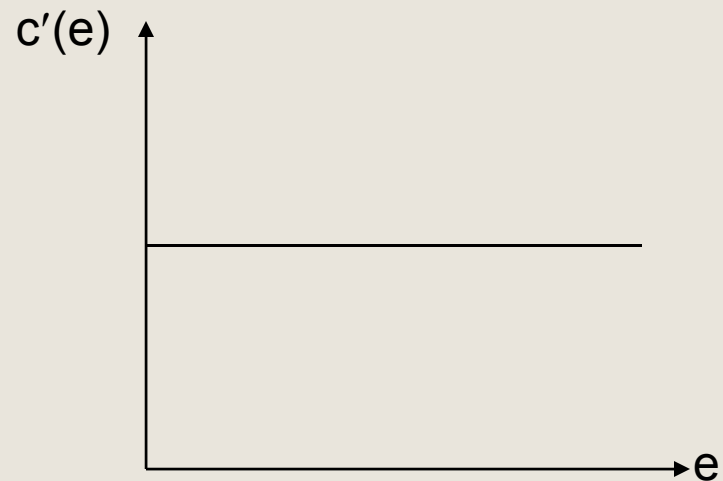
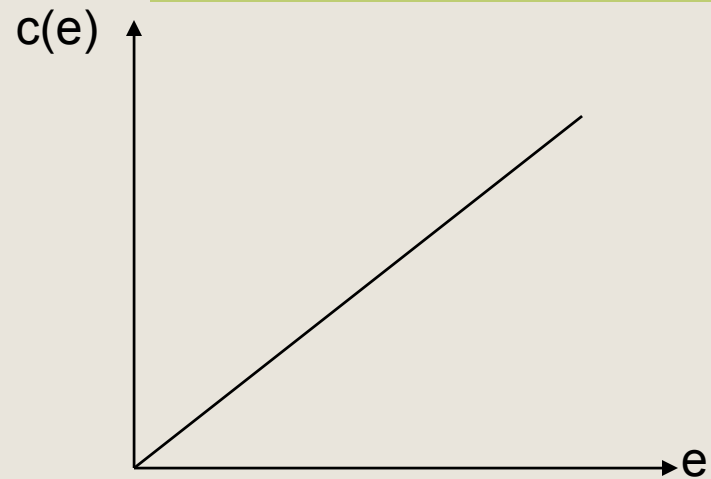
- $c(e)$ is the cost of effort function
- $c'(e) > 0$: marginal cost of effort
- $c''(e) \geq 0$: non-decreasing marginal cost of effort

- This utility function implies no wealth effects
 - Marginal cost of effort independent of salary ($U_{es} = 0$)
- More general utility function with wealth effects:
 - $U = U(s, e)$, with $U_{es} \neq 0$

Case $c', c'' > 0$, e.g. $c = e^2$



Case $c' > 0 = c''$, e.g. $c = e$



Outside Option and Participation Constraint

Outside Option (R)

- Reservation utility, or opportunity cost
 - Value of leisure if not working, OR
 - Utility from best alternative employment

Participation constraint

- Condition to ensure physician's participation:
(4) $U = s - c(e) \geq R$

Salary Model – Summary of Elements

Element	Summary
Parties	Physician, Ministry of Health
Production Technology	$q=q(e)$, with $q'>0>q''$
Information	All information is known to all parties
Payoffs	Physician: $U=s-c(e)$, outside option R Ministry: $\Pi=pq-s$, outside option 0
Contract	[salary, effort]
Timing	<ol style="list-style-type: none"> 1. Ministry designs the contract 2. Physician accepts or rejects 3. Production and Payoffs

Designing Optimal Salary Contract

The Problem

$$\begin{aligned} \text{Max}_{e,s} \Pi &= pq(e) - s \\ \text{subject to: } U &= s - c(e) \geq R \end{aligned}$$

The Solution

Step 1: $U=R$

- If indifferent, the physician will accept the offer.
- Therefore, $s=R+c(e)$.
- Therefore, $\Pi= pq(e)-c(e)-R$

2b. Salary Model – Optimal Contract

Step 2: Choose optimal e

$$\text{Max}_e pq(e)-c(e)-R$$

- First-order necessary condition:

$$pq'(e^*)-c'(e^*)=0$$

$$\Rightarrow (5) pq'(e^*)=c'(e^*)$$

Step 3: Choose optimal s

- Substitute for e^* in the participation constraint

$$\Rightarrow (6) s^*=R+c(e^*)$$

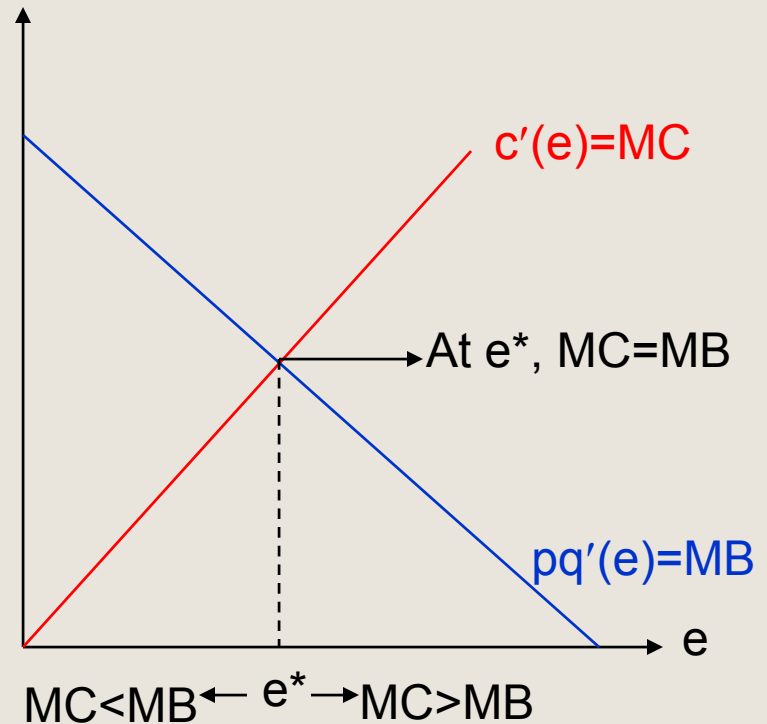
Step 4: Check that $\Pi(e^*,s^*)\geq 0 \Rightarrow pq(e^*)-c(e^*)-R\geq 0$

2b. Salary Model – Optimal Contract

What does this mean?

Optimal effort

- $pq'(e^*)=c'(e^*)$
- Marginal benefit = Marginal cost
- Marginal benefit:
 - Additional q' patients are treated, which is valued at $\$pq'$
- Marginal cost:
 - Have to increase compensation by additional $\$c'$

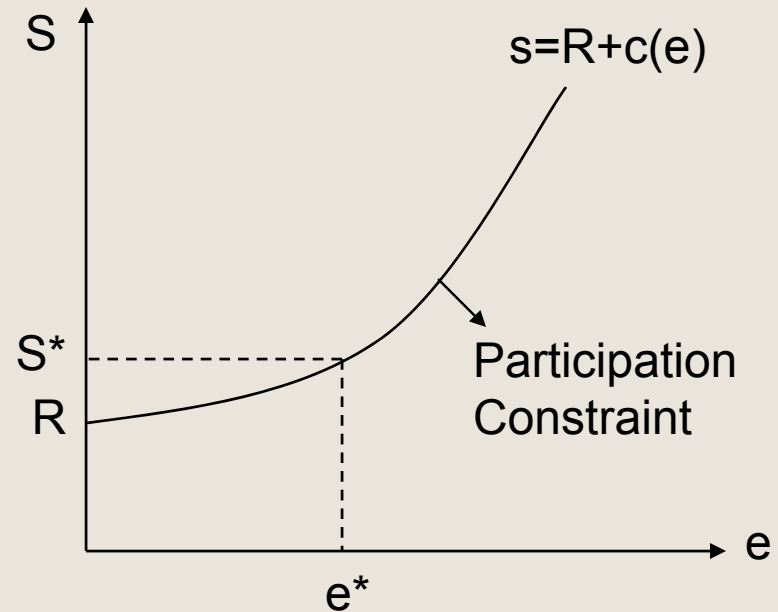


2b. Salary Model – Optimal Contract

What does this mean?

Compensation

- $s^* = R + c(e^*)$
- Compensation for:
 1. Outside option
 2. Cost of effort
- Two steps:
 1. Choose e^* : $MB(e) = MC(e)$
 2. Given e^* , choose s^* so that $s^* = R + c(e^*)$



3. Application: Salary Contract for Psychiatrists

- Approach to designing optimal salary:
 - Use $pq'(e^*)=c'(e^*)$ to determine workload
 - Use $s^*=R+c(e^*)$ to determine compensation
- Interpretation:
 - e = days of work per week
 - q = number of patients per day
 - p = value of services provided to a patient
 - R = reservation utility, e.g. if working as a physician in Alberta

3. Application: Salary for ACT Physicians

Data Requirements

- Need information on: p , R ; shape of $q(e)$ and $c(e)$
- Suppose:
 - $p = \$30/\text{patient}$
 - $R = \$1,000/\text{week}$
 - $q(e) = 20e$
 - $c(e) = \$60e^2$
- Then, the optimal salary contract is:
 - $e^* = 5 \text{ days/week}$ [$\$30 \times 20 = \$120e^*$]
 - $s^* = \$2,500/\text{week}$ [$s^* = \$1,000 + \60×5^2]

4. Efficiency and Equity

Choosing Salary Level

Effort	Salary	Utility	Profits
5 days/week	\$2,500/week	\$1,000/week	\$500/week
5 days/week	\$2,750/week	\$1,250/week	\$250/week
5 days/week	\$3,000/week	\$1,500/week	\$0/week

Efficient effort
 $MB(5)=MC(5)$

All salary levels are acceptable to both the physician and the Ministry of Health!

What is the best salary level?

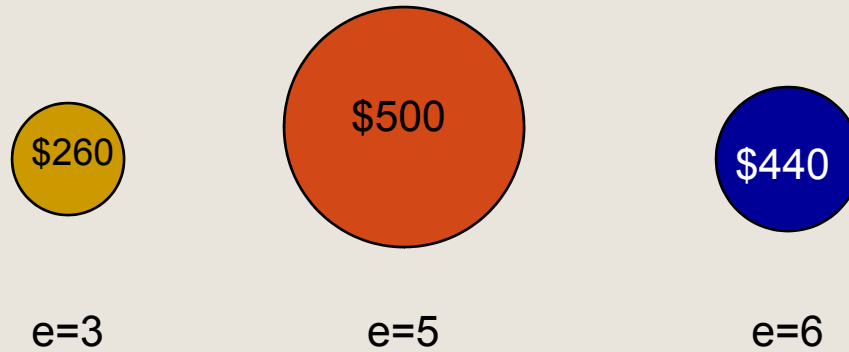
4. Efficiency and Equity

Efficiency and Equity

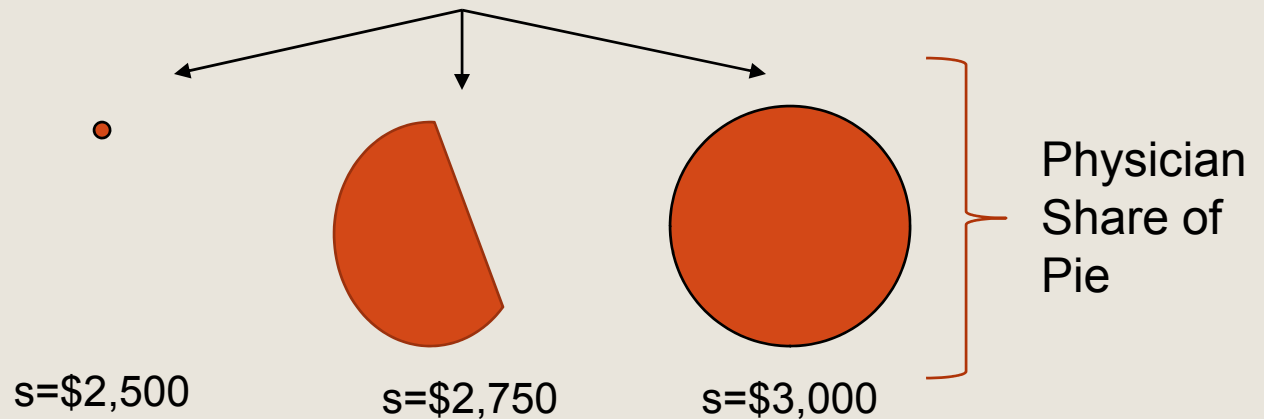
- Distinguish between:
 - Efficiency
 - Relates to level of effort
 - How to make economic pie as large as possible?
 - Economic pie = $[pq(e)-c(e)]-[R+0]$
 - Equity
 - Relates to level of compensation
 - How to divide economic pie?

Efficiency and Equity

Which pie?
(Efficiency)



How to divide pie?
(Equity)



4. Efficiency and Equity

Efficiency and Equity

- Our concern:
 - Efficiency (optimal level of effort)
 - Independent who designs the contract
- Equity
 - Depends on who designs the contract (bargaining power)
 - If the Ministry designs the contract:
 - $s^* = R + c(e^*) \Rightarrow U^* = R, \Pi^* > 0$
 - If the physician designs the contract:
 - $s^* = pq(e^*) \Rightarrow U^* > R, \Pi^* = 0$

Optimal Salary Contract with Observable Effort

- Suppose:
 - The principal can observe the agent's effort;
 - The agent's utility and principal's profit functions are $s - c(e)$ and $p q(e) - s$, respectively; and
 - The outside options for the agent and the principal are R and Π_R , respectively.
- Then, the optimal salary contract $[e^*, s^*]$ is such that:
 1. $p q'(e^*) = c'(e^*)$
 2. $R + c(e^*) \leq s^* \leq p q(e^*) - \Pi_R$